UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

Homework #3 - MATH 271 - L01 & L02

Follow instructions available in the Assignment Policy document!

Question 1

a: Prove in full detail that no function from a finite set X to a strictly larger finite set Y is onto.

Now for $k \in \mathbb{N}$, define a function $f : X \to Y$ to be "$k - to - 1$" if $N(f^{-1}(y)) \leq k$ for each $y \in Y$.

b: Prove the *Generalized pigeonhole principle*, that is if $k \in \mathbb{N}$, X and Y are finite sets such that $N(X) > k \cdot N(Y)$, then no function from X to Y is $k - to - 1$.

c: Given finite sets X and Y such that $N(X) > N(Y)$, describe explicitly how to compute the smallest $k \in \mathbb{N}$ such that no function from X to Y is $k - to - 1$, and prove that this is indeed the smallest such k.

Question 2 Argue the following by either providing a detailed proof or counterexample.

Consider two functions $f : X \to Y$ and $g : Y \to X$ where X and Y are two finite sets, and assume further that

$$g \circ f = 1_X$$

a: Does necessarily $f \circ g = 1_Y$?

b: Is g necessarily one-one?

c: Is f necessarily one-one?

d: Is g necessarily onto?

e: Is f necessarily onto?

Question 3

a: Show that for any set A of six positive integers taken from $\{1, 2, ..., 12\}$, A must contain two disjoint subsets whose elements when added up give the same sum.

b: What about any set of five positive integers taken from $\{1, 2, ..., 12\}$?