Faculty of Science<br>Department of Mathematics \& Statistics

## Homework \#4 - MATH 271 - L01 \& L02

## Follow instructions available in the Assignment Policy document!

## Question 1

a: Let $a, b$ and $c$ be integers. Prove that if $\operatorname{gcd}(a, b)=1, a \mid c$ and $b \mid c$, then $a b \mid c$.
b: Show that the assumption that $\operatorname{gcd}(a, b)=1$ in part (a) is necessary.

## Question 2

a: Prove Theorem 10.4.3 part 4.
That is let $a, b$ and $n$ be integers with $n>1$ and $a \equiv b(\bmod n)$. Prove by induction that $a^{m} \equiv b^{m}(\bmod n)$ for all integers $m \geq 1$.
(You may use the other parts of the Theorem in your proof).
b : Argue the following statement:
Let $a, b, c, d$ and $n$ be all non-negative integers with $n>1$ and such that $a \equiv b(\bmod n)$ and $c \equiv d(\bmod n)$. Then $a^{c} \equiv b^{d}(\bmod n)$.

## Question 3

a: With justification, find an inverse for 3276 modulo 3025.
b: With justification, find an inverse for 3276 modulo 3026.
c: You have intercepted the encrypted message $C=8$ which you know has been encrypted using the RSA cipher using the public key $p q=1271$ and $e=43$. With justification, what is the message $M$ ?

