Course Outline
University of Calgary
Faculty of Science & Werklund School of Education
Bachelor of Education, Winter 2019
MATH 305 - (EDUC 305) Inside Mathematics

Lauren DeDieu
Department of Mathematics and Statistics
Email: lauren.dedieu@ucalgary.ca
Phone: 403 220 5056
Office Location: MS 528

Paulino Preciado
Werklund School of Education
Email: apprecia@ucalgary.ca
Phone: 403 220 5277
Office Location: EDT 834

Lectures: **Monday, Wednesday, and Friday, 11:00 - 11:50, ST 143**
Tutorial 1: Wednesdays 14:00 – 14:50, SB 148
Tutorial 2: Thursdays 12:00 – 12:50, SB 124A

Start date: Thursday, January 10.
End date: Friday, April 12.
Term break (no class): February 17 to 24

Course Overview

Through an exploration of the usually-tacit elements of mathematical concepts and processes, the course focuses on strategies for unpacking concepts and for sustained engagement in inquiry. This course may not be repeated for credit.

This course will be co-taught by scholars from the Faculty of Science and Werklund School of Education. Tutorials are offered to support students with the mathematical content related to the course.

Pre-requisites: Mathematics 211 or 213; and 271 or 273.

Learning Outcomes

By the end of this course, students should have increased facility with

- analyzing mathematical concepts – in particular, being able to identify associations (e.g., metaphors, images, exemplars) that render concepts comprehensible and useful;
- explicating the role of context in the emergence of mathematical concepts – that is, appreciating the role of cultural circumstances in enabling and constraining the development of mathematical insights; and
- asking mathematical questions – that is, sustaining engagement in mathematical inquiry.

Elaborated Description

For centuries, both mathematical inquiry and mathematics learning have been assumed to be principally logical. However, recent studies of the processes of human cognition and the nature of mathematical insight have revealed that mathematics learning and mathematics research are

highly analogical. Moreover, the specific analogies that are made available can either help or hinder the development of mathematical knowledge – a point that is true on individual, social, and cultural levels.

Oriented by that realization, this course is concerned with what lurks "inside" mathematical concepts and processes. To explain, the work of mathematicians is often characterized in terms of converting ideas into highly condensed representations, in large part to facilitate further mathematical exploration. However, while these condensed formulations enable more powerful mathematical thought, they can present unique challenges to learners. In particular, much of mathematics learning must be about unpacking or decompressing concepts – that is, separating and then reblending the elements that mathematicians have assembled into comprehensible and useful constructs.

This course focuses mainly on strategies for packing and unpacking concepts, and it situates these strategies within mathematical inquiry. The course has three intertwining emphases:

1) Concept Study

Concept study involves tracing the associations that render a concept meaningful. It can involve examinations of the origins and applications of a concept, explorations of the representations (e.g., metaphors, images, exemplars) used to describe it, and surveys of other concepts in its mathematical neighborhood. Concept study is focused in particular on the analogical aspects of mathematics concepts – for two reasons. Firstly, analogies are the principal mechanism of human thought, and so being attentive to these associations can aid understanding and insight. Secondly, analogies always bring along unwanted baggage, and so being aware when thinking is analogical (versus logical) can be useful for avoiding unwarranted generalizations.

This emphasis of the course will be developed through instructor-led studies of concepts that include: number, limits, and functions.

2) Cultural Framing of Mathematics

There is a popular belief that mathematical knowledge is culture free. However, when considered historically, the field has clearly evolved with society, affecting and affected by popular beliefs and assumptions, political climates, technological possibilities, and other contextual factors.

Maintaining the course focus on the concepts of number, limits, and function, this emphasis of the course will be developed by examining a few key cultural shifts (e.g., the sudden rejection of a prevailing metaphor, or a new blending of multiple instantiations) that opened up new mathematical horizons.

3) Mathematical question asking

"Mathematics," for most people, is about finding answers – whether by following formal procedures or engaging in more flexible problem solving.

Among research mathematicians, however, the enterprise is not so oriented toward end points. Rather, mathematical research is typically more about keeping the inquiry going. New insights always open up new questions. The following are among the activities that are commonly invoked to sustain mathematical inquiry:

- making conjectures,
- making and refining definitions,

- hypothesis testing and modeling,
- extending and generalizing, and
- justifying, validating, and proving

This emphasis of the course will be developed through sustained engagement with mathematical problems associated with the themes of the collective concept studies (i.e., number, limits, and functions).

The Expectation of Excellence in Professional Work

Please review the Academic Calendar carefully. It describes the program and provides detailed schedules and important dates. It contains information on expectations for student work and professional conduct. In addition, procedures are described regarding concern about student performance in the program. **Please pay especially careful attention to details and descriptions in the following topic areas:**

- ***The Importance of Attendance and Participation in Every Class***

As this is a professional program, experiences are designed with the expectation that all members will be fully involved in all classes and in all coursework experiences. As you are a member of a learning community your contribution is vital and highly valued, just as it will be when you take on the professional responsibilities of being a teacher. We expect that you will not be absent from class with the exception of documented instances of personal or family illness or for religious requirements.

- ***Engagement in class discussion and inquiry***

Another reason for the importance of attendance and participation in every class is that the course involves working with fellow students to share ideas and thinking. For example, each week you will work with a small group to engage fellow students in discussions on work being considered in class. You will also help other groups by providing ideas for scholarly inquiry in assignments. **If you find that you are experiencing difficulties as a group collaborating, please inform the instructor.**

In order to be successful in this class, you are required to do all of the readings, attend class regularly, participate in discussions and activities, and complete all assignments.

Assignments

Assignment 1 - Concept Study: Group Inquiry Project, 65%

Sub-Assignment 1: Representations/Instantiations, 15% (Due February 8)
Sub-Assignment 2: Shifting through Interpretations, 15% (Due February 25)
Peer Feedback: 10% (Due March 1)
Report: 25% (Due March 11)

This group-based project is focused on unpacking/decompressing/deconstructing a mathematical concept, selected by each group, following the instructor-led examples that will be undertaken in class. The task will comprise two sub-assignments and a final culminating product. That final product will be in a format that is accessible to all class members.

Sub-Assignment 1: Representations/Instantiations (Due in Week 4)

How might the concept be represented? What sorts of images are used to introduce and illustrate it? What sorts of metaphors are invoked to explain it? What other concepts are closely related to it? How/when did the concept arise and evolve?

Sub-Assignment 2: Sifting through Interpretations (Due in Week 6)

Working with (and possibly extending) your list of representations from the first sub-assignment, critically examine the entries. How do different interpretations channel thinking? How do they enable and constrain thinking? Which seem to afford greater mathematical power? Might some instantiations be blended into more powerful constructs?

Peer feedback (Due week 6)

During this week students will provide peer feedback on sub-assignment 2 to at least one team. A report of the feedback will be submitted for evaluation. The feedback will consist of comprehensive and detailed descriptions of the strengths and aspects to improve.

Concept Study Report (Due in Week 8)

Extending the work of the two sub-assignments, develop an integrated presentation of your investigation and make it available to your classmates through some engaging format (e.g., podcast, interactive webpage, app). An aspect of your presentation should be an illustration of the significance of your results through specific applications of the concept, following examples presented in class.

Grading Criteria for Assignment 1

Sub-Assignments 1 and 2 (15% of the final grade, each) will be graded based on how students respond to the questions included in the description of each sub-assignment.

> *Outstanding (A+ to A).* All the questions describing the sub-assignment are comprehensively and eloquently answered, with proper references (consistent to a commonly used format, such as APA) to the consulted sources, and images are carefully selected to convey key meanings of the selected mathematical concept.
>
> *Good (A- to B+).* All the questions describing the sub-assignment are answered, referencing the consulted sources, and images convey key meanings of the selected mathematical concept.
>
> *Satisfactory (B to C+).* All the questions describing the sub-assignment are addressed, some references to the consulted sources may be missing, and images do not convey key meanings of the selected mathematical concept.

Peer feedback (10% of the final grade) will be graded based on the feedback provided to other groups.

> *Outstanding (A+).* Strong feedback is provided to more than three groups.
>
> *Excellent – Strong feedback (A).* Comprehensive and detailed descriptions of the strengths and aspects to improve are given to one group. Sound suggestions for the improvement are elaborated including a rationale for each one.

Good – Relevant feedback (A- to B+). At least one strength and one aspect to improve are addressed including suggestions for improvement with explanations of such suggestions.

Satisfactory (B to C+). One strength and one aspect to improve are addressed including suggestions for improvement with superficial or no explanation of such suggestions.

The Concept Study Report (25% of the final grade) will be graded in terms of both content and presentation. Presentation quality includes: Format and organization, Spoken and written English, use of visuals, and proper citations.

Outstanding (A+ to A). The report is an accurate and comprehensive, yet succinct, elaboration of the selected mathematics concept addressing images, metaphors, analogies of the concept (and related concepts), and considering cultural, historical and societal features. Presentation is clear, eloquent and well written (or spoken), using consistently a common format for referencing sources (such as APA).

Good (A- to B+). The report is an accurate and thoughtful elaboration of the selected mathematics concept addressing images, metaphors, analogies of the concept (and related concepts), and considering cultural, historical and societal features. Presentation is clear and well written (or spoken), using a common format for referencing sources (such as APA).

Satisfactory (B to C+). The report is an elaboration of the selected mathematics concept, superficially addressing images, metaphors, analogies of the concept (and related concepts), and considering some cultural, historical, societal features. Presentation is clear and includes references for consulted sources.

Assignment 2 - Question Asking, 25%, (Individual)

Preliminary Report: 5% (Due March 25)
Final Report: 20% (Due April 12)

This individual assignment involves engaging in, self-monitoring of, and reporting on a mathematical inquiry. It will begin with a problem posed by the instructor.

However, for this assignment, the problem is not the question and its solution is not the answer. That is, while your inquiry may involve solving the problem, the more substantial part of the task is to keep the inquiry going by asking new questions. For example, you might ask simpler versions of the problem, inquire into extensions, seek generalizations of aspects of your solution, contrive related problems, or prove insights associated with your solution. And so on.

Preliminary Report (Due in week 10)

Students will receive feedback from the instructor(s) on this report.

Final Report (Due in week 12)

Your submission will be a hybrid of narrative and mathematical reporting. Narrative elements should address key decision-making moments, provide insight into your thinking processes, identify obstacles and other challenges, speak to strategies used, and so on – in essence, tracking and classifying the sorts of questions you posed as you moved through your inquiry. Where

appropriate, it should also highlight key moments of associative thinking, such as uses of images or analogies, that advanced or hindered your thinking.

Grading Criteria for Assignment 2

The Preliminary Report (5% of final grade) will be graded in terms of completion. It is enough to submit the report for obtaining a full mark.

The Final Report (20% of final grade) will be graded based on the engagement in inquiry on a problem posed by the teacher.

> *Outstanding (A+ to A).* Solutions (or partial solutions) to the original question are presented using different representations. Extensions to these solutions are discussed, including generalizations of the original problem. Decision-making processes are well described, including the evolution of questions that furthered the inquiry. References are properly cited using a common format (such as APA).
>
> *Good (A- to B+).* A solution (or partial solution) to the original question is presented using different representations. Extensions to this solution are discussed, including generalizations of the original problem. The decision-making process is documented, including questions that furthered the inquiry.
>
> *Satisfactory (B to C+).* A solution (or partial solution) to the original question is included. At least one extension to the solutions is discussed, including a generalization of the original problem. The decision-making process is mentioned, including at least one question furthering the inquiry.

Assignment 3: Regular in-class tasks, 10% (Due each class)

Lecture sessions will include tasks on a regular basis.

Grading Criteria for Assignment 3

This task will be assessed by completion. The grade will be proportional to the engagement in the class tasks during the course.

Schedule of weekly activities/readings (topics and activities subject to change)

Topics/Themes	Readings and Assignments
Week 0: January 10 - 11 **Introduction to the course**	Read Chapters 1 to 3 of Mazur (2004)
Week 1: January 14 - 18 **Concept Study** Complex numbers and polynomial equations	Read Chapters 4 to 9 of Mazur (2004)
Week 2: January 21 - 25 **Concept Study**	Read Chapters 10 to 12 of Mazur (2004)

Topics/Themes	Readings and Assignments
Number: Historical, cultural snapshots of confusion and breakthroughs	
Week 3: January 28 - February 1 **Concept Study** Logic construction of number systems	Read Appendix A of D'Angelo and west (2000).
Week 4: February 4 - 8 **Concept Study** Area and Limits: Historical, cultural snapshots of confusion and breakthrough	Investigate the historical and cultural understanding of the concept of limit, including its advances to measurement. Estimating areas of shapes **Sub-assignment 1: Due February 8**
Week 5: February 11 - 15 **Concept Study** Areas and Limits: A contemporary narrative and future open possibilities	Compute Riemman and Lebesgue integrals and calculate areas of surfaces.
Week of February 18 - 22	**Break, No classes**
Week 6: February 25 - March 1 **Concept Study** Functions: Representations/Instantiations, interpretations	Identify several representations and instantiations of a function and discuss merits and limitations. **Sub-assignment 2: Due February 25** **Peer feedback: Due March 1**
Week 7: March 4 - 8 **Concept Study** Functions: Contemporary narrative and varieties	Application of common mathematical functions used in mathematics, including their particular representations.
Week 8: March 11 - 15 **Concept Study** Mathematical Thinking: Specialization, Generalization, Extension	Read and address the problems in Chapters 1 and 2 from Mason, Burton and Stacey (1982/2010). **Assignment 1 Report: Due March 11**
Week 9: March 18 - 22 **Question Asking** Mathematical Thinking: Conjecturing and testing	Read chapters 3 to 4 from Mason, Burton and Stacey (1982/2010). Engage in selected mathematical problems from Mason, Burton and Stacey (1982/2010) and Hamilton (2013).
Week 10: March 25 - 29 **Question Asking** On the question of What do we know?	Review of key mathematical branches and the types od questions they address. **Assignment 2 Preliminary Report: Due March 25**
Week 11: April 1 - 5 **Question Asking** Justifying, Convicting, Proving, Questioning	Read chapters 5 to 8 from Mason, Burton and Stacey (1982/2010). Continue the mathematical exploration with a focus on justifying, convicting, proving and questioning.
Week 12: April 8 - 12 **Question Asking**	Work on the Question Asking assignment. **Assignment 2 Final Report: Due April 12**

Required Texts:

D'Angelo, J., & West, D. (2000). *Mathematical thinking: Problem solving and proofs, 2nd ed.*, Prentice Hall. (Only Appendix A: will be provided through D2L).

Mazur, B. (2004). *Imagining numbers (particularly the square root of minus fifteen*). New York: Penguin books.

Mason, J., Burton, L., & Stacey, K. (2010). Thinking mathematically, 2nd ed. Prentice Hall. **(The first edition, 1982, could be used, too).**

Hamilton, G. (2013) *$1,000,000 unsolved problems for k to 12* http://mathpickle.com/wp-content/uploads/2016/01/Unsolved-K-12-winners.pdf

Recommended References:

Cajori, F. (1980). *A history of mathematics.* 3d ed. New York: Chelsea Pub. Co.

Du Sautoy, M. (2010). A brief history of mathematics. BBC podcast. http://www.bbc.co.uk/podcasts/series/maths

Burton, D. (2010) The history of mathematics: An introduction, 7th ed. McGraw-Hill.

Martinez, A. A. (2006). Chapter 3. History: Much ado about less than nothing. In A. A. Martinez, *Negative math: How mathematical roles can be positively bent*, (pp. 18-42). Princeton, NJ: Princeton University Press.

Mazur, J. (2014). *Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers.* Princeton, NJ, USA: Princeton University Press. Available online through the library.

Tao, T. (2006). *Solving mathematical problems: A personal perspective*. Oxford University Press.

Zames, F. (2008). *Surface area and the cylinder area paradox*. Mathematical Association of America. http://www.maa.org/programs/maa-awards/writing-awards/surface-area-and-the-cylinder-area-paradox

Grading scheme

Grade	GPA Value	%	Description per U of C Calendar
A+	4.0	95-100	Outstanding
A	4.0	90-94	Excellent – Superior performance showing comprehensive understanding of the subject matter
A-	3.7	85-89	
B+	3.3	80-84	
B	3.0	75-79	Good - clearly above average performance with knowledge of subject matter generally complete
B-	2.7	70-74	
C+	2.3	65-69	
C	2.0	60-64	Satisfactory - basic understanding of the subject matter
C-	1.7	55-59	

D+	1.3	52-54	Minimal pass - Marginal performance
D	1.0	50-51	
F	0.0	49 and lower	Fail - Unsatisfactory performance

Students in the BEd must have an overall GPA of 2.5 in the semester to continue in the program without repeating courses.

It is strongly recommended that students complete and submit each assignment on or before the day it is due. Late work will be accepted without penalty only if special arrangements are made with the instructor prior to the assignment due date. Students must pass each assignment in order to successfully complete the course. Writing proficiency will be considered in the assessment of the assignments.

Academic Accommodation: Students with a disability, who require academic accommodation, need to register with **Student Accessibility Services** http://www.ucalgary.ca/access/ located in MSC 452, Telephone: 210-6019. Academic accommodation letters need to be provided to course instructors no later than fourteen (14) days after the first day of class. ***It is a student's responsibility to register with the Student Accessibility Services and to request academic accommodation, if required.*** Students who have not registered with Student Accessibility Services are not eligible for formal academic accommodation.

BB.1.1 Accommodations on Protected Grounds other than Disability
Students who require an accommodation in relation to their coursework or to fulfil requirements for a graduate degree, based on a protected ground other than disability, should communicate this need, preferably in writing, to their Instructor or the appropriate Associate Dean, Department Head or the department/faculty designated contact person. Students who require an accommodation unrelated to their coursework or the requirements for a graduate degree, based on a protected ground other than disability, should communicate this need, preferably in writing, to the Vice-Provost (Student Experience). For additional information on support services and accommodations for students with disabilities, **visit www.ucalgary.ca/access/**

The Freedom of Information Protection of Privacy Act prevents instructors from placing assignments or examinations in a public place for pickup and prevents students from access to exams or assignments other than their own. Therefore, students and instructors may use one of the following options: return/collect assignments during class time or during instructors' office hours, students provide instructors with a self-addressed stamped envelope, or submit/return assignments as electronic files attached to private e-mail messages.

Surveys
At the University of Calgary, feedback through the Universal Student Ratings of Instruction (USRI) survey and the Faculty of Science Teaching Feedback form provides valuable information to help with evaluating instruction, enhancing learning and teaching, and selecting courses. Your responses make a difference - please participate in these surveys.

Copyright of Course Materials

All course materials (including those posted on the course D2L site, a course website, or used in any teaching activity such as (but not limited to) examinations, quizzes, assignments, laboratory manuals, lecture slides or lecture materials and other course notes) are protected by law. These materials are for the sole use of students registered in this course and must not be redistributed. Sharing these materials with anyone else would be a breach of the terms and conditions governing student access to D2L, as well as a violation of the copyright in these materials, and may be pursued as a case of student academic or non-academic misconduct, in addition to any other remedies available at law.

Intellectual honesty/Plagiarism:
Intellectual honesty is the cornerstone of the development and acquisition of knowledge and requires that the contribution of others be acknowledged. As a result, cheating or plagiarism on any assignment or examination is regarded as an extremely serious academic offence, the penalty for which may be an F on the assignment and possibly also an F in the course, academic probation, or requirement to withdraw.
The University of Calgary Calendar states that plagiarism involves submitting or presenting work as if it were the student's own work when it is not. Any ideas or materials taken from another source written, electronic, or oral must be fully and formally acknowledged.
Plagiarism includes but is not limited to:
(a) The work submitted or presented was done, in whole or in part, by an individual other than the one submitting or presenting the work (this includes having another impersonate the student or otherwise substituting the work of another for one's own in an examination or test),
(b) Parts of the work are taken from another source without reference to the original author,
(c) The whole work (e.g., an essay) is copied from another source, and/or,
(d) A student submits or presents work in one course which has also been submitted in another course (although it may be completely original with that student) without the knowledge of or prior agreement of the instructor involved.
While it is recognized that scholarly work often involves reference to the ideas, data and conclusions of other scholars, intellectual honesty requires that such references be explicitly and clearly noted. Plagiarism is an extremely serious academic offence. Plagiarism occurs not only when direct quotations are taken form a source without specific acknowledgement, but also when original ideas or data form the source are not acknowledged. A bibliography is insufficient to establish which portions of the students' work are taken form external sources; footnotes or other recognized forms of citation must be used for this purpose.

Cheating is an extremely serious academic offence. Cheating at tests or examinations includes but is not limited to dishonest or attempted dishonest conduct such as speaking to other candidates or communicating with them under any circumstances whatsoever; bringing into the examination room any textbook, notebook, memorandum, other written material or mechanical or electronic device not authorized by the examiner; writing an examination or part of it, or consulting any person or materials outside the confines of the examination room without permission to do so, or leaving answer papers exposed to view, or persistent attempts to read other students' examination papers.

Other Academic Misconduct includes, but is not limited to, tampering or attempts to tamper with examination scripts, class work, grades and/or class records; failure to abide by directions by an instructor regarding the individuality of work handed in; the acquisition,

attempted acquisition, possession, and/or distribution of examination materials or information not authorized by the instructor; the impersonation of another student in an examination or other class assignment; the falsification or fabrication of clinical or laboratory reports; the non-authorized tape recording of lectures.
Any student who voluntarily and consciously aids another student in the commission of one of these offences is also guilty of academic misconduct.

Mental Health

The University of Calgary recognizes the pivotal role that student mental health plays in physical health, social connectedness and academic success, and aspires to create a caring and supportive campus community where individuals can freely talk about mental health and receive supports when needed. We encourage you to explore the excellent mental health resources available throughout the university community, such as counselling, self-help resources, peer support or skills-building available through the SU Wellness Centre (Room 370, MacEwan Student Centre, https://www.ucalgary.ca/wellnesscentre/services/mental-health-services) and the Campus Mental Health Strategy website (http://www.ucalgary.ca/mentalhealth/).

Sexual violence

The University of Calgary is committed to fostering a safe, productive learning environment. The Sexual Violence Policy (https://www.ucalgary.ca/policies/files/policies/sexual-violence-policy.pdf) is a fundamental element in creating and sustaining a safer campus environment for all community members. We understand that sexual violence can undermine students' academic success and we encourage students who have experienced some form of sexual misconduct to talk to someone about their experience, so they can get the support they need. The Sexual Violence Support Advocate, Carla Bertsch, can provide confidential support and information regarding sexual violence to all members of the university community. Carla can be reached by email (svsa@ucalgary.ca) or phone at 403-220-2208.

Emergency Evacuation/Assembly Points: In case of emergency during class time, be sure to FAMILIARIZE YOURSELF with the information on assembly points.

Safewalk: Promoting Campus Safety and Awareness: Twenty four hours a day, seven days a week, Safewalk volunteers walk people safely to their destination on campus. This service is free and available to students, staff and campus visitors. Safewalks are done in male/female pairs. The volunteers walk anywhere on campus (including McMahon Stadium, Health Sciences, Student Family Housing, the Alberta Children's Hospital and the University LRT station). To request a Safewalk volunteer to walk with you,

- Call 403-220-5333 (24 hours a day/seven days a week, 365 days a year)
- Use the Help Phones (they are not just for emergencies)
- Approach an on-duty Safewalker and request a walk.

Education Students Association (ESA) President for 2018 – 2019 is **Sam Sirianni,** smsirian@ucalgary.ca, esa@ucalgary.ca

Student Ombuds Office:
http://www.ucalgary.ca/ombuds/

Werklund SU Representative, 2018 - 2019 is **Tina Miller**, educrep@su.ucalgary.ca
Society of Undergraduate Mathematics - (SUM-C), 2018 – 2019 representative is **Selena Iwaskow**, sumc@ucalgary.ca
SU Faculty of Science Rep: Phone: 403-220-3913 Email: sciencerep@su.ucalgary.ca