AB

MATH 305 / EDUC 305: Inside Mathematics

Winter, 2024

Károly Bezdek
Department of Mathematics and Statistics
Email: *kbezdek@ucalgary.ca*
Phone: *403 220-6919*

Paulino Preciado
Werklund School of Education
Email: *apprecia@ucalgary.ca*
Phone: *403 220 5277*

Office Hours: *By appointment only*

Lectures: *Monday, Wednesday, Friday, 11:00 - 11:50, HNSC 128*
Tutorial 1: *Wednesdays 14:00 – 14:50, EEEL 445*
Tutorial 2: *Thursdays 12:30 – 13:20, EEEL 445*

Important Dates:

- *Start date: Monday, January 8*
- *End date: Tuesday, April 9*
- *Term break (no class): February 18 to 24*
- *Last day to drop a class without financial penalty: January 18*
- *Last day to add or swap a course: January 19*
- *Last day to withdraw from a course: April 9*

Statutory Holidays:

- *Alberta Family Day: Monday, February 19*
- *Good Friday: Friday, March 29*
- *Easter Monday: Monday, April 1*

Pre-requisites: Mathematics 211 or 213; and 271 or 273.

Email: Students are required to use a University of Calgary (@ucalgary.ca) email address for all correspondence.

COURSE DESCRIPTION:

Through an exploration of the usually-tacit elements of mathematical concepts and processes, the course focuses on strategies for unpacking concepts and for sustained engagement in inquiry.
This course may not be repeated for credit.
This course will be co-taught by scholars from the Faculty of Science and Werklund School of Education.
Tutorials are offered to support students with the content related to the course.
Pre-requisites: Mathematics 211 or 213; and 271 or 273.

LEARNER OUTCOMES:

By the end of this course, students are expected to be able to

- analyzing mathematical concepts identifying associations (e.g., metaphors, images, exemplars) that render concepts comprehensible and useful;

- investigating the role of context (natural, social, cultural, political, and historical) in the emergence of mathematical concepts to formulate an explanation of how cultural circumstances enable and constrain the development of mathematical insights;
- asking mathematical questions that sustain engagement in mathematical inquiry; and
- communicating mathematical ideas and arguments accurately to diverse audiences (e.g. school students, adults and mathematicians).

ELABORATED DESCRIPTION

For centuries, both mathematical inquiry and mathematics learning have been assumed to be principally logical. However, recent studies of the processes of human cognition and the nature of mathematical insight have revealed that mathematics learning and mathematics research are highly analogical. Moreover, the specific analogies that are made available can either help or hinder the development of mathematical knowledge – a point that is true on individual, social, and cultural levels.

Oriented by that realization, this course is concerned with what lurks "inside" mathematical concepts and processes. To explain, the work of mathematicians is often characterized in terms of converting ideas into highly condensed representations, in large part to facilitate further mathematical exploration. However, while these condensed formulations enable more powerful mathematical thought, they can present unique challenges to learners. In particular, much of mathematics learning must be about unpacking or decompressing concepts – that is, separating and then reblending the elements that mathematicians have assembled into comprehensible and useful constructs.

This course focuses mainly on strategies for packing and unpacking concepts, and it situates these strategies within mathematical inquiry. The course has three intertwining emphases:

1) Concept Study

Concept study involves tracing the associations that render a concept meaningful. It can involve examinations of the origins and applications of a concept, explorations of the representations (e.g., metaphors, images, exemplars) used to describe it, and surveys of other concepts in its mathematical neighborhood. Concept study is focused in particular on the analogical aspects of mathematics concepts – for two reasons. Firstly, analogies are the principal mechanism of human thought, and so being attentive to these associations can aid understanding and insight. Secondly, analogies always bring along unwanted baggage, and so being aware when thinking is analogical (versus logical) can be useful for avoiding unwarranted generalizations.

This emphasis of the course will be developed through instructor-led studies of concepts that include: number, limits, and functions.

2) Cultural Framing of Mathematics

There is a popular belief that mathematical knowledge is culture free. However, when considered historically, the field has clearly evolved with society, affecting and affected by popular beliefs and assumptions, political climates, technological possibilities, and other contextual factors.

Maintaining the course focus on the concepts of number, limits, and function, this emphasis of the course will be developed by examining a few key cultural shifts (e.g., the sudden rejection of a prevailing metaphor, or a new blending of multiple instantiations) that opened up new mathematical horizons.

3) Mathematical question asking

"Mathematics," for most people, is about finding answers – whether by following formal procedures or engaging in more flexible problem solving.

Among research mathematicians, however, the enterprise is not so oriented toward end points. Rather, mathematical research is typically more about keeping the inquiry going. New insights always open up new questions. The following are among the activities that are commonly invoked to sustain mathematical inquiry:

- making conjectures,
- making and refining definitions,
- hypothesis testing and modeling,

- extending and generalizing, and
- justifying, validating, and proving

This emphasis of the course will be developed through sustained engagement with mathematical problems associated with the themes of the collective concept studies (i.e., number, limits, and functions).

COURSE DESIGN AND DELIVERY:

This course will be delivered face-to-face on campus and includes engagement in a D2L environment.

REQUIRED RESOURCES:

Andreescu, T., & Andrica, D. (2014). *Complex Numbers from A to ... Z* (2nd ed. 2014.). Birkhäuser Boston. https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/46l39d/alma991028055306304336

Mason, J., Burton, L., & Stacy, K. (2010). Thinking Mathematically (2nd Edition, Chapters 1 to 4). New York: Prentice Hall. First two chapter available online:
https://www.pearsonhighered.com/assets/samplechapter/m/a/s/o/Mason - Chapter 1.pdf
https://www.pearsonhighered.com/assets/samplechapter/m/a/s/o/Mason - Chapter 2.pdf
Full text available in e-book format for instant download at Amazon Kindle:
https://www.amazon.ca/dp/B08H3XW9J4?ref=KC_GS_GB_CA

Mazur, B. (2004). *Imagining numbers (particularly the square root of minus fifteen*). New York: Penguin books. https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/46l39d/alma991024789749704336
Amazon Kindle ; Apple ; Google Play Books ; Kobo ; ebooks.com

Núñez, R., & Marghetis T. (2014). *Cognitive Linguistics and the Concept (s) of Number*. In R. C. Kadosh & A. Dowker (Eds.), *The Oxford Handbook of Numerical Cognition* (pp. 377 – 401). Oxford, UK: Oxford University Press. https://ebookcentral-proquest-com.ezproxy.lib.ucalgary.ca/lib/ucalgary-ebooks/reader.action?docID=2095058&ppg=408

ADDITIONAL RESOURCES:

Cajori, F. (1980). *A history of mathematics.* 3d ed. New York: Chelsea Pub. Co.
https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/1rf6mu5/alma991020051699704336

D'Angelo, J., & West, D. (2000). *Mathematical thinking: Problem solving and proofs, 2nd ed.*, Prentice Hall. (Only Appendix A: will be provided through D2L).
https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/46l39d/alma991017185559704336

Du Sautoy, M. (2010). A brief history of mathematics. BBC podcast.
http://www.bbc.co.uk/podcasts/series/maths

Burton, D. (2010) The history of mathematics: An introduction, 7th ed. McGraw-Hill.
https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/1rf6mu5/alma991028153887904336

Hamilton, G. (2013) *$1,000,000 unsolved problems for k to 12* http://mathpickle.com/wp-content/uploads/2016/01/Unsolved-K-12-winners.pdf

Lakoff, G. & Núñez, R. (2000). Where mathematics come from. New York, NY: Basic Books.
https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/46l39d/alma991002732969704336

Martinez, A. A. (2006). Chapter 3. History: Much ado about less than nothing. In A. A. Martinez, *Negative math: How mathematical roles can be positively bent*, (pp. 18-42). Princeton, NJ: Princeton University

Press. https://ebookcentral-proquest-com.ezproxy.lib.ucalgary.ca/lib/ucalgary-ebooks/reader.action?docID=5675276&ppg=31

Mazur, J. (2014). *Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers.* Princeton, NJ, USA: Princeton University Press. Available online through the library. https://www-degruyter-com.ezproxy.lib.ucalgary.ca/document/doi/10.1515/9781400850112/html

Tao, T. (2006). *Solving mathematical problems: A personal perspective*. Oxford University Press. https://ucalgary.primo.exlibrisgroup.com/permalink/01UCALG_INST/46l39d/alma991012887259704336

Zames, F. (2008). *Surface area and the cylinder area paradox*. Mathematical Association of America. http://www.maa.org/programs/maa-awards/writing-awards/surface-area-and-the-cylinder-area-paradox
https://www-jstor-org.ezproxy.lib.ucalgary.ca/stable/3026930

LEARNING TASKS OVERVIEW

LEARNING TASK	DESCRIPTION OF LEARNING TASK	GROUP / INDIVIDUAL	WEIGHT	DUE DATE
LT1. Concept Study	Complex Numbers: Concept Study Draft: Concept Study Final Report: Concept Study	Group (two members)	10% 5% 20%	Feb. 16 Mar 8 Apr. 9
LT 2. Mathematical question asking	Draft: Conjectures & Proofs Presentation: Conjectures & Proofs	Group (two members)	5% 30%	Fri. Mar. 15 Mar. 25-Apr. 8
LT 3. Course Engagement	In-Class/Tutorial/Out-of-Class Activities	Individual/ Group	15%	Ongoing
LT 4. Tests	Test 1 (Complex Numbers) Test 2 (Constructing Number Systems) Test 3 (Functions and Limits)	Individual	5% 5% 5%	Feb. 2 Mar 1 Mar 22

WEEKLY COURSE SCHEDULE:

Date	Topic	Readings and Tutorials	Due Dates
Week 1: Jan. 8-12	Complex numbers	Chapter 1 from Andreescu & Andrica (2014) **Tutorial:** Complex Numbers	
Week 2: Jan. 15-19	Complex numbers	Chapter 1 from Andreescu & Andrica (2014) **Tutorial:** Complex Numbers	
Week 3: Jan 22-26	Complex numbers	Chapter 2 from Andreescu & Andrica (2014) Chapter 1-3 from Mazur (2004) **Tutorial:** Complex Numbers	
Week 4: Jan. 29 – Feb. 2	Complex numbers Number: Historical, cultural snapshots of confusion and breakthroughs	Chapter 2 from Andreescu & Andrica (2014) **Tutorial:** Complex Numbers Chapters 4-7 from Mazur (2004)	**F:** Test 1 (LT 4)
Week 5: Feb. 5-9	Concept Study	Chapters 8-10 from Mazur (2004) **Tutorial:** Concept Study drop-in	
Week 6: Feb. 12-16	Concept Study	Chapters 10-12 from Mazur (2004) **Tutorial:** Concept Study drop-in	**F:** Complex Numbers: Concept Study (LT 1)
Term Break			
Week 7: Feb. 26 – Mar. 1	Constructing Number Systems	**Tutorial:** Constructing Number systems **Tutorial:** Concept Study drop-in	**F:** Test 2 (LT 4)
Week 8: Mar. 4-8	Mathematical Thinking: Specialization, Generalization and Extension	**W:** Read chapters 1 to 2 from Mason, et al. (1982/2010). **Tutorial:** Concept Study drop-in	**F:** Draft: Concept Study (LT 1)
Week 9: Mar. 11-15	Mathematical Thinking: Conjecturing and testing	**W:** Read chapters 3 to 4 from Mason, et al. (1982/2010). **Tutorial:** Conjecture & Proof drop-in	**F:** Draft: Conjecture & Proof (LT 2)
Week 10: Mar. 18-22	Functions and Limits	**Tutorial:** Functions and Limits **Tutorial:** Concept Study drop-in	**F:** Test 3 (LT 4)
Week 11: Mar. 25-29	Mathematical Question Asking	Conjectures & Proofs Presentations (Class & Tutorial)	**Presentations:** Conjectures & Proofs LT 2
Week 12: Apr. 1-5	Mathematical Question Asking	Conjectures & Proofs Presentations (Class & Tutorial)	
Week 13: Apr. 8	Wrap-up		**T:** Final Report: Concept Study (LT 1)

CHANGES TO SCHEDULE:

Please note that changes to the schedule may occur to meet the emerging needs and dynamics of the participants in the course.

LEARNING TASKS AND ASSESSMENT

There are four required Learning Tasks for this course.

1. **LEARNING TASK 1: CONCEPT STUDY: 35%.**

   - **Complex Numbers: Concept Study**, 10% (Fri. Feb. 16th)
   - **Draft: Concept Study**, 5% (Fri. March 8th)
   - **Final Report: Concept Study**, 20% (Tu. Apr. 9nd)

   This group-based (two members) project will focus on the following two themes:

   Theme 1. Representations/Instantiations:
   How might the concept be represented? What sorts of images are used to introduce and illustrate it? What sorts of metaphors are invoked to explain it? What other concepts are closely related to it? How/when did the concept arise and evolve?

   Theme 2. Sifting through Interpretations:
   Working with (and possibly extending) your list of representations from the first sub-assignment, critically examine the entries. How do different interpretations channel thinking? How do they enable and constrain thinking? Which seem to afford greater mathematical power? Might some instantiations be blended into more powerful constructs?

   In the *Complex Numbers: Concept Study*, groups will address these two themes using the concept of complex numbers.
   In the *Concept Study Draft*, groups will choose their own mathematical concept and work to unpack/decompress/deconstruct this concept by focusing on these two themes.
   After receiving feedback, groups will work to improve this draft and submit a final *Concept Study*.

   CRITERIA FOR ASSESSMENT OF LEARNING TASK 1

   The *Group Project* will be graded based on how students respond to the questions included in the two themes described above (see *Representations/Instantiations, Sifting through Interpretations*). Questions must be comprehensively and eloquently answered, with proper references to the consulted source. Images should be carefully selected to convey key meanings of the selected mathematical concept. The submission must demonstrate a mastery of mathematical content and include a critical analysis which shows depth. The piece should be succinct and include personal conclusions which eloquently synthesize the material. The written submissions and presentation should be clear and well written (or spoken).

2. **LEARNING TASK 2: MATHEMATICAL QUESTION ASKING: 35%**

   - **Draft: Conjectures & Proofs**, 5% (Fri. Mar. 15th)
   - **Presentation: Conjectures & Proofs**, 30% (Mar. 25 – Apr. 8th in class/tutorial. Dates will be assigned to each group after the groups are chosen.)

   This learning task involves engaging in, self-monitoring of, and reporting on a mathematical inquiry. It will begin with a problem posed by the instructor. It will be completed in groups (two members).

   Conjectures & Proofs: For this assignment, the problem is not the question and its solution is not the answer. That is, while your inquiry may involve solving the problem, the more substantial part of the task is to keep the inquiry going by asking new questions. For example, you might ask simpler versions of the problem, inquire into extensions, seek generalizations of aspects of your

solution, contrive related problems, or prove insights associated with your solution. And so on.
Draft: Students will receive feedback from the TA on this report.
Presentation: Your presentation will be a hybrid of narrative and mathematical reporting. Narrative elements should address key decision-making moments, provide insight into your thinking processes, identify obstacles and other challenges, speak to strategies used, and so on – in essence, tracking and classifying the sorts of questions you posed as you moved through your inquiry. Where appropriate, it should also highlight key moments of associative thinking, such as uses of images
Mathematical Modelling Report: In this assignment, your "client" will pose a problem and you will create a model that represents this problem. Your written report will describe your model to your client, specify any assumptions you are making and any limitations to the model, and will offer your recommendation. Results should be communicated in a way that would be easy for the "client" to understand.

CRITERIA FOR ASSESSMENT OF LEARNING TASK 2

Draft (5%): will be graded based on engagement and not according to mathematical correctness. If you submit a draft that is complete, addresses all aspects of the prompt, and demonstrates a reasonable amount of effort, then you can expect to receive full marks.
Presentation (30%): will be graded based on the engagement in inquiry on a problem posed by the instructor. Solutions (or partial solutions) to the original question must be presented using different representations. Extensions to these solutions should be discussed, including generalizations of the original problem. Decision-making processes should be well described, including the evolution of questions that furthered the inquiry. The presentation should be engaging, precise, and clear.

3. **LEARNING TASK 3: COURSE ENGAGEMENT: 15%** (In-Class Activities/ Tutorial Activities/ Out-of-Class Activities)
The course component will assess content covered in the course textbook and in class. They will be assigned by the instructors as the course progresses. Students are expected to attend class and may not receive advanced warning about graded in-class activities. Students will receive advanced notice when a graded activity will occur in tutorial.

CRITERIA FOR ASSESSMENT OF LEARNING TASK 3

These activities will be graded in terms of engagement (e.g., active participation in class, addresses the prompts, and quality of presentation). Further details about grading criteria will be provided in class.

4. **LEARNING TASK 4: TESTS: 15%**
   - **Test 1**, 5% (Feb. 2nd)
   - **Test 2**, 5% (Mar. 1st)
   - **Test 3**, 5% (Mar. 22nd)

The course component will assess content covered in the course.

CRITERIA FOR ASSESSMENT OF LEARNING TASK 4

Each test will be evaluated in terms of appropriate answers. Further details about grading criteria will be provided in class.

THE EXPECTATION OF EXCELLENCE IN PROFESSIONAL WORK

Please review the Academic Calendar carefully. It describes the program and provides detailed schedules and important dates. It contains information on expectations for student work and professional conduct. In addition, procedures are described regarding concern about student performance in the program. Please pay especially careful attention to details and descriptions in the following topic areas:

- *The Importance of Attendance and Participation in Every Class*

As this is a professional program, experiences are designed with the expectation that all members will be fully involved in all classes and in all coursework experiences. As you are a member of a learning community your contribution is vital and highly valued, just as it will be when you take on the professional responsibilities of being a teacher. We expect that you will not be absent from class with the exception of documented instances of personal or family illness or for religious requirements.

- *Engagement in Class Discussion and Inquiry*

Another reason for the importance of attendance and participation in every class is that the course involves working with fellow students to share ideas and thinking. For example, each class you will work with a small group to engage fellow students in discussions on work being considered in class. You will also help other groups by providing ideas for scholarly inquiry in assignments. **If you find that you are experiencing difficulties as a group collaborating, please inform the instructor.**

In order to be successful in this class, you are required to do all of the readings, attend class regularly, participate in discussions and activities, and complete all assignments.

You may be invited to participate in research involved in this course. However, the instructors will not know whether you will be participating in the research until the end of the course, when grades have been submitted.

EXPECTATIONS FOR WRITING

All written assignments (including, to a lesser extent, written exam responses) will be assessed at least partly on writing skills. Writing skills include not only surface correctness (grammar, punctuation, sentence structure, etc.) but also general clarity and organization. Sources used in research papers must be properly documented. If you need help with your writing, you may use the writing support services in the Learning Commons. For further information, please refer to the official online University of Calgary Calendar, Academic Regulations, E. Course Information, E.2: Writing Across the Curriculum: http://www.ucalgary.ca/pubs/calendar/current/e-2.html

LATE SUBMISSIONS

All late submissions of assignments must be discussed with the instructor **prior to the due date.** Students may be required to provide written documentation of extenuating circumstances (e.g. statutory declaration, doctor's note, note from the University of Calgary Wellness Centre, obituary notice); a penalization for late submission might be imposed. A deferral of up to 30 days may be granted at the discretion of the Associate Dean of Undergraduate Programs with accompanying written evidence.

ISSUES WITH GROUP TASKS

With respect to group work, if your group is having difficulty collaborating effectively, please contact the instructor immediately. If a group is unable to collaborate effectively or discuss course materials online in a timely manner, the instructor may re-assign members to different groups or assign individual work for completion.

GRADING

Grade	GPA Value	%	Description per U of C Calendar
A+	4.0	95-100	Outstanding
A	4.0	90-94	Excellent – Superior performance showing comprehensive understanding of the subject matter
A-	3.7	85-89	
B+	3.3	80-84	
B	3.0	75-79	Good - clearly above average performance with knowledge of subject matter generally complete
B-	2.7	70-74	
C+	2.3	65-69	
C	2.0	60-64	Satisfactory - basic understanding of the subject matter
C-	1.7	55-59	
D+	1.3	52-54	Minimal pass - Marginal performance
D	1.0	50-51	
F	0.0	49 and lower	Fail - Unsatisfactory performance

Students in the B.Ed. program must have an overall GPA of 2.5 in the semester to continue in the program without repeating courses.

Academic Accommodation
It is the student's responsibility to request academic accommodations according to the University policies and procedures listed below. The student accommodation policy can be found at: https://www.ucalgary.ca/legal-services/sites/default/files/teams/1/Policies-Student-Accommodation-Policy.pdf. Students needing an accommodation because of a disability or medical condition should communicate this need to Student Accessibility Services in accordance with the Procedure for Accommodations for Students with Disabilities: ucalgary.ca/legal-services/sites/default/files/teams/1/Policies-Accommodation-for-Students-with-Disabilities-Procedure.pdf. Students needing an accommodation in relation to their coursework based on a Protected Ground other than Disability, should communicate this need, preferably in writing, to their Instructor.

Academic Misconduct
For information on academic misconduct and its consequences, please see the University of Calgary Calendar at http://www.ucalgary.ca/pubs/calendar/current/k.html

Attendance/ Prolonged Absence
Students may be asked to provide supporting documentation for an exemption/special request. This may include, but is not limited to, a prolonged absence from a course where participation is required, a missed course assessment, a deferred examination, or an appeal. Students are encouraged to submit documentation that will support their situation. Supporting documentation may be dependent on the reason noted in their personal statement/explanation provided to explain their situation. This could be medical certificate/documentation, references, police reports, invitation letter, third party letter of support or a statutory declaration etc. The decision to provide supporting documentation that best suits the situation is at the discretion of the student.

Falsification of any supporting documentation will be taken very seriously and may result in disciplinary action through the Academic Discipline regulations or the Student Non-Academic Misconduct policy.

https://www.ucalgary.ca/pubs/calendar/current/n-1.html

The Freedom of Information Protection of Privacy Act prevents instructors from placing assignments or examinations in a public place for pickup and prevents students from access to exams or assignments other than their own. Therefore, students and instructors may use one of the following options: return/collect assignments during class time or during instructors' office hours, students provide instructors with a self-addressed stamped envelope, or submit/return assignments as electronic files attached to private e-mail messages.

For additional resources including, but not limited to, those aimed at wellness and mental health, student success or to connect with the Student Ombuds Office, please visit https://www.ucalgary.ca/registrar/registration/course-outlines

Education Students Association (ESA) President for the academic year is Claire Gillis, esa@ucalgary.ca.

Werklund SU Representative is Elsa Stokes, educrep@su.ucalgary.ca.