Department of Mathematics and Statistics
University of Calgary

Math 311 Sheet 1 Fall 2007

1. Use mathematical induction to prove each of the following:

a. $1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{1}{3} n (n+1)(n+2)$
for all $n \geq 1$

b. $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots n(n+1)(n+2) = \frac{1}{4} n (n+1)(n+2)(n+3)$
for all $n \geq 1$

c. $1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$
for all $n \geq 1$

d. $1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2 = \frac{1}{6} n (n+1)(2n+1)$
for all $n \geq 1$

e. $1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 + n^3 = \left[\frac{n(n+1)}{2}\right]^2$
for all $n \geq 1$

f. $1^4 + 2^4 + 3^4 + \cdots + (n-1)^4 + n^4 = \frac{1}{30} n (n+1)(2n+1)\left(3n^2 + 3n - 1\right)$
for all $n \geq 1$

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2. Use mathematical induction to show that:

   a. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{(n-1)^2} + \frac{1}{n^2} \leq 2 - \frac{1}{n}$
   for all $n \geq 1$

   b. $n^2 \leq 2^n$ *for all* $n \geq 4$

3. Give a conjecture for the following sum and then prove that your conjecture is true.

   $1 \cdot 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5 + \cdots n(n+1)(n+2)(n+3)$

   for all $n \geq 1$

4. Suppose that A, B, and P are all square matrices, P being invertible, and that $A = P^{-1}BP$. Use mathematical induction to show that

   $A^n = P^{-1}B^nP$ *for all* $n \geq 1$.

5. If A is a non-singular matrix, prove that A^t is also a non-singular matrix.

6. If A and B are non-zero nxn matrices with the property that $AB = 0$. Prove that neither A nor B is invertible.

7. Describe all the 2x2 matrices with the property that $A^2 = 0$.

8. Describe all the 2x2 matrices with the property that $A^2 = I$.