Department of Mathematics and Statistics
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Math 311 Sheet 3

1. Give an example of a set together with addition and scalar multiplication by real numbers defined such that the structure is not a vector space. Be able to explain why you can make this assertion.

2. Consider the set, V, consisting of all integers together with zero. Define the binary operation of $\circ$ as follows:
$For \quad a, b, \in V \qquad a \circ b = a + b - ab$
Determine whether or not the operation $\circ$ is commutative or associative. Determine whether or not the there is an additive identity element. Justify your answer in each case.

3. a. Consider the vector space, $P_3 = \{ f(x) : degree \quad of \quad f(x) \leq 3 \}$. Let $U = \{f(x) : degree \quad of \quad f(x) = 2\}$. Determine whether or not U is a subspace of P_3. Justify your answer.

   b. Consider the vector space, M_{22}. Let $U = \{ A \in M_{22} : A \text{ is invertible} \}$. Determine whether or not U is a subspace of M_{22}. Justify your answer.

4. a. Show that $M_{22} = span \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right\}$.

   b. Let V be a vector space. Suppose that $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$. Show that
   i. $span \{ \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \} = span \{ \boldsymbol{u}, \boldsymbol{u} + \boldsymbol{v}, \boldsymbol{v} + \boldsymbol{w} \}$
   ii. $span \{ \boldsymbol{u}, \boldsymbol{v} \} = span \{ \boldsymbol{u} + 3\boldsymbol{v}, \boldsymbol{u} - 2\boldsymbol{v} \}$
   iii. $span \{ \boldsymbol{u} + \boldsymbol{v} + \boldsymbol{w}, \boldsymbol{v} + \boldsymbol{w}, \boldsymbol{w} \} = span \{ \boldsymbol{u} + 3\boldsymbol{v}, \boldsymbol{u} - 2\boldsymbol{v}, 2\boldsymbol{u} - \boldsymbol{v} + 2\boldsymbol{w} \}$

5. Exercise 6.2, questions 19, to 27.