Math 311 Sheet 4

1. Let $A = \begin{pmatrix} 2 & 5 & 9 \\ 1 & 3 & 2 \end{pmatrix}$. Consider the homogeneous system of equations given by $A\underline{X} = \{\bar{0}$. Determine the set, U of solutions for this system. Show that U is a subspace of $\mathbb{R}^3$ and determine the dimension and basis for U.

Definitions

Given an $m \times n$ matrix A, we define

(i) col A = column space of A is the subspace of $\mathbb{R}^m$ spanned by the columns of A.

(ii) row A = row space of A is the subspace of $\mathbb{R}^n$ spanned by the rows of A.

(iii) null A = null space of A = $\{ \underline{X} \in \mathbb{R}^n : A\underline{X} = \bar{0} \}$.

(iv) Im A = $\{ A\underline{X} : \underline{X} \in \mathbb{R}^n \}$ = Image space of A.

2. Given $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 0 & 1 \\ 4 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

Determine the basis and dimension of
(1) col A
(2) row A
(3) null A
(4) Im A

Let R be the row echelon form of A. Determine the dimension and basis for row R.

3. Let $A = \begin{pmatrix} 3 & 1 \\ 2 & 0 \\ 4 & 2 \end{pmatrix}$, $V = \begin{pmatrix} 2 & 8 \\ 3 & 12 \end{pmatrix}$ and $W = \begin{pmatrix} 2 & 7 \\ 3 & 10 \end{pmatrix}$

Determine $\operatorname{col}(A)$, $\operatorname{col}(AV)$ and $\operatorname{col}(AW)$

Verify that (i) $\operatorname{col}(AV) \subseteq \operatorname{col} A$

(ii) $\operatorname{col}(AW) = \operatorname{col} A$.

4. U and W are ~~sub~~ subspaces of a v.s. V. If U and W have finite dimension,

(i) Show that $U+W$ is a subspace of V when

$U + W = \{\vec{u} + \vec{w} : \vec{u} \in U \text{ and } \vec{w} \in W\}$.

4. (ii) $\dim(U+W) = \dim U + \dim W - \dim U \cap W$.