

Department of Mathematics and Statistics
University of Calgary
Sheet 3

Math 311

1. Give an example of a set together with addition and scalar multiplication by real numbers defined such that the structure is not a vector space. Be able to explain why you can make this assertion.

2. Consider the set, V , consisting of all integers together with zero. Define the binary operation of \circ as follows:

$$\text{For } a, b, \in V \quad a \circ b = a + b - ab$$

Determine whether or not the operation \circ is commutative or associative. Determine whether or not there is an additive identity element. Justify your answer in each case.

3. a. Consider the vector space, $P_3 = \{f(x) : \text{degree of } f(x) \leq 3\}$. Let $U = \{f(x) : \text{degree of } f(x) = 2\}$. Determine whether or not U is a subspace of P_3 . Justify your answer.

b. Consider the vector space, M_{22} . Let $U = \{A \in M_{22} : A \text{ is invertible}\}$. Determine whether or not U is a subspace of M_{22} . Justify your answer.

4. a. Show that $M_{22} = \text{span} \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right\}$.

b. Let V be a vector space. Suppose that $u, v, w \in V$. Show that

i. $\text{span} \{u, v, w\} = \text{span} \{u, u + v, v + w\}$

ii. $\text{span} \{u, v\} = \text{span} \{u + 3v, u - 2v\}$

iii.

$$\text{span} \{u + v + w, v + w, w\} = \text{span} \{u + 3v, u - 2v, 2u - v + 2w\}$$

5. Exercise 6.2, questions 19, to 27.