Department of Mathematics and Statistics
University of Calgary

Math 311

Sheet 4

[Linear Independence, Basis and Dimension]

1. Determine whether or not the following sets of vectors are linearly independent in the vector space given.

   a. In the vector space $\mathbb{R}^4$ under normal addition and scalar multiplication,

   $$\boldsymbol{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \end{pmatrix}.$$

   b. In the vector space $\mathbb{R}^4$, under the usual addition and scalar multiplication,

   $$\boldsymbol{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 4 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 2 \\ 3 \\ 3 \\ 6 \end{pmatrix}.$$

2. Determine whether or not the following sets of vectors form a basis for the given vector space. If the given set of vectors is not a basis for the vector space given, determine a basis for the subspace generated by these vectors and extend that basis to a basis for the vector space given.

   a. $\boldsymbol{u} = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ in the vector space M_{22}.

   b. $\boldsymbol{u} = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ in the vector space M_{22}.

3. Determine all values of x so that the following set of vectors in $\mathbb{R}^4$ is linearly independent.

   a. Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ x \end{pmatrix}, \quad \boldsymbol{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}, \quad \boldsymbol{w} = \begin{pmatrix} 0 \\ 1 \\ 4 \\ 2 \end{pmatrix}, \quad \boldsymbol{q} = \begin{pmatrix} 1 \\ 3 \\ 1 \\ 3 \end{pmatrix}.$

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b. Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ x \\ 5 \\ 2 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 2 \\ 1 \\ x \\ 1 \end{pmatrix}$.

Exercise 6.3, Questions 1 to 32.
Exercise 6.4, Questions 1 to 19