# MATHEMATICS 311 L02 WINTER 2005
# ADDITIONAL EXERCISES

**1**. Find *null* $A$ and *im* $A$ for the following matrices:

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 2 & 2 \end{bmatrix}$.

(b) $A = \begin{bmatrix} -1 & 3 & 1 \\ 2 & 2 & 2 \\ 3 & 1 & 2 \end{bmatrix}$.

**2**. Let $X_1 = [1, -1, 1]^T$ and $X_2 = [1, 1, 1]^T$. For each of the following, determine whether $X \in span\,\{X_1,\ X_2\}$.
(a) $X = [-1, -5, -1]^T$.
(b) $X = [2, 0, -2]^T$.

**3**. Determine whether each of the following sets are linearly independent.
(a) $\left\{[1,1,1]^T,\ [1,-1,1]^T,\ [1,-2,4]^T\right\}$.
(b) $\left\{[-1,3,1]^T,\ [1,1,1]^T,\ [3,1,2]^T\right\}$.

**4**. Let $M = \left\{[-1,3,1]^T,\ [1,1,1]^T,\ [3,1,2]^T, [1,5,2]^T\right\}$
(a) Verify that $M$ spans $\mathbb{R}^3$.
(b) Find a subset of $M$ that is a basis of $\mathbb{R}^3$.

**5**. Let $U = \left\{[x,y,z]^T \in \mathbb{R}^3 \mid x+y+z=0\right\}$ (in other words, U is the plane through the origin with normal $N = [1,1,1]^T$)
(a) Show that $U$ is a subspace of $\mathbb{R}^3$.
(b) Find a basis of $U$.
(c) Verify that $[1,-2,1]^T \in U$ and find a basis of $U$ containing $[1,-2,1]^T$.

**6**. Let $\mathcal{F} = \{F_1, F_2\}$ and $\mathcal{G} = \{G_1, G_2\}$ where $F_1 = [2,1]^T$, $F_2 = [3,2]^T$, $G_1 = [-2,1]^T$ and $G_2 = [3,-1]^T$.
(a) Let $X = [1,1]^T$. Find $C_{\mathcal{F}}(X)$ and $C_{\mathcal{G}}(X)$.
(b) Given that $C_{\mathcal{F}}(Y) = [1,-2]^T$. Find $Y$ and $C_{\mathcal{G}}(Y)$.
(c) Let $\mathcal{E}$ be the standard basis of $\mathbb{R}^2$. Find $P_{\mathcal{F}\leftarrow\mathcal{E}}$, and $P_{\mathcal{G}\leftarrow\mathcal{E}}$.
(d) Find $P_{\mathcal{G}\leftarrow\mathcal{F}}$.
(e) Let $T$ be a linear operator on $\mathbb{R}^2$ with $M_{\mathcal{F}}[T] = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$. Find $T\left([1,1]^T\right)$, $T\left([1,0]^T\right)$, $T\left([0,1]^T\right)$ using the properties of linear transformations, and find the standard matrix of $T$.
(f) Is the operator $T$ in part (e) invertible? If so, find $M_{\mathcal{F}}[T^{-1}]$.
(g) For the operator $T$ in part (e), find $M_{\mathcal{G}}[T]$.

(h) Let $S$ be a linear operator on $\mathbb{R}^2$ with $M_{\mathcal{F}}[S] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$. Let $T$ be a linear operator on $\mathbb{R}^2$ in part (e). Find $M_{\mathcal{G}}[S]$, $M_{\mathcal{F}}[T \circ S]$ and $M_{\mathcal{F}}[S \circ T]$