Faculty of Science
Department of Mathematics & Statistics

FINAL EXAMINATION - MATH 311 - L02

December 13, 2000

Your family name: ________________________

Your first name: ________________________

Your signature: ________________________

Your student number: ________________________

INSTRUCTIONS:

I: Fill out the above information BEFORE starting this exam.
II: Read the following instructions very carefully.
III: Show all your work.
IV: Use the back of the previous page for rough work.
V: Write a **clear** solution in the answer space provided.
VI: No calculator or other material allowed.
VII: There are 7 questions and 7 pages to this exam.
VIII: Time allowed is 3 hours.

PROBLEM	#1	#2	#3	#4	#5	#6	#7	TOTAL
MARKS	/5	/10	/10	/5	/5	/10	/5	/50

Question 1 (5 points)

[1] a) Define what it means for a matrix A to be orthogonal, and for a linear transformation T to preserve distances.

[4] b) Show that if T a linear operator on $\mathbb{R}^n$ given by $T(X) = AX$, where A is an orthogonal matrix, then T preserves distances.

Question 2 (10 points)
[2] a) Given a basis $\mathcal{F} = \{X_1, \cdots, X_n\}$ of $\mathbb{R}^n$ and a linear operator T on $\mathbb{R}^n$, describe the **role** of the $\mathcal{F}$-matrix $\mathcal{M}_{\mathcal{F}}(T)$ of the operator T, and how to **compute** it.

[5] b) Consider the operator T on $\mathbb{R}^3$ given by the reflection in the plane P given by the equation $x + y + z = 0$. Find a basis $\mathcal{F}$ such that $\mathcal{M}_{\mathcal{F}}(T)$ is diagonal, and compute that matrix.

[3] c) Use part a) and b) to compute $T(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$.

Question 3 (10 points)
Answer the following questions by TRUE or FALSE; if you answer FALSE, you must provide a numerical example supporting your claim:

[2] a) An orthogonal set of 4 vectors in $\mathbb{R}^4$ is always a spanning set of $\mathbb{R}^4$.

Example (If necessary): ANSWER:________

[2] b) The columns of **any** 3×4 matrix are linearly dependent.

Example (If necessary): ANSWER:________

[2] c) $null(A^T) = imA$ for any matrix A.

Example (If necessary): ANSWER:________

[2] d) Every basis of $\mathbb{M}_{2,2}$ contains an invertible matrix.

Example (If necessary): ANSWER:________

[2] e) If an $n \times n$ matrix A has an orthogonal set of n eigenvectors, then A has an orthonormal set of n eigenvectors.

Example (If necessary): ANSWER:________

Question 4 (5 points)
Show that eigenvectors X and Y of a symmetric matrix A corresponding to different eigenvalues λ and μ respectively must be orthogonal.
[Hint: Evaluate the expression $(\lambda - \mu)X \bullet Y$]

Question 5 (5 points)

[2] a) Show that $U = \{A \text{ in } \mathbb{M}_{2,2} |\ A \text{ is symmetric } \}$ is a subspace of $\mathbb{M}_{2,2}$.

[3] b) Find a basis of U.

Question 6 (10 points)
A computer graphics specialist would like to display a 3-dimensional object by simply projecting it on a flat screen identified by the plane P with equation $x + y + z = 0$.

[5] a) If the point $A(1, 2, 3)$ is on the object, then find the point B on the screen where it will be displayed.

[5] b) Find the point A on the plane $z = 2$ that is displayed on the screen at the location $B(1, -2, 1)$.

Question 7 (5 points)
Argue the following statement:

$$(rowA)^{\perp} = nullA \text{ for any matrix } A.$$