

# MATH 311

## BONUS QUESTIONS I

For credit on a bonus question turn in a written sheet with your name, course number, ID, and the solution - including a full explanation or proof. It is assumed that your work on a bonus question is done on the honour system, that is, by yourself and with no help from others or from books/Internet. Each successful solution is worth 2 quiz points. Have fun, due date for this set is February 27. The answers must be clearly written and will be graded strictly, but second attempts are allowed and your solution can also be discussed with the instructor and/or TA. 😊!

1. Four friends have to cross a river and have a boat that holds two people. Each friend has a certain crossing time, and these times are 1,2,5,10 minutes. The boat will take the time of the slower person to cross, for example if the 2 and 5 persons go in the boat the crossing will take 5 minutes. Find out how they can get all four of them across the river in total time just 17 minutes.

2. Determine the next row in the sequence:

1  
11  
21  
1211  
111221  
312211

3. A young lady named Eva is invited to a barbecue at a friend's house. When she arrives her friend greets her and she remarks "What a cute bunch of children playing in your garden." Her friend says "Yes, they actually belong to four families. My family is the largest, my brother's smaller, my sister's smaller yet, and my cousin's smallest of all. They wanted to play baseball but there were not enough to make up two teams, so they are playing a few other games. Curiously, the product of the numbers of children in the four families equals my house number." Eva then says "I happen to be a big fan of Sherlock Holmes, so since I know your house number let me see if I can figure out how many children are in each family." She does some calculations and finally says "Hmm, I'm getting close but could you kindly answer one more question: does your cousin's family consist of just a single child?" Her host answers this question, and then Eva tells just how many children are in each family.

Question: How many children are in each family?

4. Solve the anagram  $SEND + MORE = MONEY$ . Here each letter stands for a distinct digit, and one has to fill in the letter so that it makes sense as an addition problem. Also note that the first letters of the numbers, S and M, cannot equal 0. There is a unique solution, and to receive the bonus you should not only find the solution but explain (prove) that it is the only possible solution.
5. You are given 12 coins, which look identical except one of them is counterfeit. You are also given a balance (not a scale, just a balance).
  - (a) Warm-up question : Given that the counterfeit coin is heavier, find it using the balance just 3 times (don't turn this one in, only (b)).
  - (b) Bonus question : Find the counterfeit coin using the balance just 3 times, knowing in advance only that it has a different weight from the others, but not knowing if it is lighter or heavier.
6. Explain how the magic trick involving three unknown numbers was done in the lecture. To recall the trick, a volunteer wrote three numbers, integers between 0 and 10, on the blackboard. The sum was taken and circled. Then 12 times the first number was taken, to this was added the second number, the result multiplied by 10, and finally the circled number was added to this. The total was given to the instructor, who then was able to find all three original numbers.