MATH 311 MAPLE ASSIGNMENT

Detailed instructions for completing this assignment will be given on a separate sheet called MAPLETIPS, and using these the assignment should take an hour or less. Only stapled assignments will be accepted, non-stapled assignments go into the garbage. Questions should be numbered and done in order. Due date is March 25.

ASSIGNMENT

1. Your name on top of first page, and ID number on top of second page.

2. (a) Find π to 100 digits.

   (b) Determine the 100th digit of π (the 1st is 3, 2nd 1, 3rd 4, etc.)

3. (a) Find the zeros (roots) of the polynomial $p(x) = x^3 - 5x^2 + 7x - 13$.

   (b) Which formula was used by MAPLE in solving (A)?

   (c) Evaluate the zeros found in (a) to 30 digits.

4. Consider the following three vectors in $\mathbb{R}^4$: $\mathbf{u}_1 = [2,1,2,0]^T, \mathbf{u}_2 = [0,1,-1,2]^T, \mathbf{u}_3 = [2,0,0,1]^T$. Find three orthonormal vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ having the same span as $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

   In the following questions consider the matrices

$$A = \begin{bmatrix} 1 & -2 & 2 & 3 & 0 & 4 \\ 4 & 4 & -1 & 7 & 2 & -5 \\ 2 & 3 & 5 & 9 & 1 & 2 \\ 0 & 2 & 3 & 9 & 3 & 1 \\ 7 & -6 & 0 & 15 & 4 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & -1 & 3 & 0 & 4 \\ -4 & 1 & 2 & 5 & -1 \\ -2 & 0 & 7 & 2 & 0 \\ 9 & -1 & 3 & -13 & 6 \\ -11 & -1 & 3 & 0 & 13 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 5 & 0 \\ 4 & 0 & -2 \end{bmatrix}.$$

5. (a) State a property of the matrix C, and because of this property what can you say about the eigenvalues of C. [Hint : see 5.5 Exercise 24 or p.452, The Spectral Theorem (a).]

   (b) Find the eigenvalues of C to 30 digit accuracy.

   (c) The answers in (b) will be complex numbers. Why is this not a contradiction to the Spectral Theorem?

6. (a) Find rank(A).

   (b) Find the RREF of A. How many pivots are there, and compare this with your answer in (a).

   (c) Find BA.

7. (a) Find $\det(B)$.

   (b) Is B invertible? Explain your answer

   (c) If B is invertible, find B^{-1}.

8. Find the eigenvalues of B, and find an eigenvector for the eigenvalue $\lambda = -13$.

9. Consider the stochastic matrix

$$P = \begin{bmatrix} .3 & .2 & 0 & .4 \\ .2 & 0 & .1 & 0 \\ .1 & .3 & .8 & .5 \\ .4 & .5 & .1 & .1 \end{bmatrix}.$$

   (a) Compute P^2, P^5.

   (b) Is P a regular stochastic matrix? Explain.

   (c) Compute P^{10}, P^{50}, and use this to estimate the steady state vector for P.

10. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 1 & 4 & 2 \\ 1 & 0 & -3 & 1 \\ 4 & -2 & -3 & 6 \end{bmatrix}.$$

(a) Find the eigenvalues and eigenvectors of A.

(b) Explain why A is diagonalizable.

(c) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

(d) Use MAPLE to verify that $P^{-1}AP$ is really diagonal.