# ASSIGNMENT 2.

**MATH 311** **FALL 2010**

1. (a) *Show that* $\mathbb{P}_3 = span\{x^2+2,\ x+1,\ x^3-2x,\ 5x\}$. 2 marks

   (b) *Suppose* $A$ *and* $B$ *are nonzero* $n \times n$ *matrices. If* $A^T = A$ *and* $B^T = -B$*, show that* $\{A, B\}$ *is an independent subset of* $\mathbb{M}_{nn}$. 3 marks

   (c) *If* $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ *is an independent set in a vector space* $V$*, show that* $\{\mathbf{u}+\mathbf{v},\ \mathbf{v}+\mathbf{w},\ \mathbf{w}+\mathbf{u}\}$ *is also independent.* 3 marks

2. *Let* $U$ *and* $W$ *be subspaces of a vector space* $V$*, and assume that* $dim(U) = 2$*. Show that either* $U \subseteq W$ *or* $dim(U \cap W) \leq 1$. 8 marks

3. *Let* $A$ *be an* $m \times n$ *matrix.*

   (a) *If* $rank(A) = n$ *and* $AB = 0$ *where* $B$ *is* $n \times k$*, show that* $B = 0$. 4 marks

   (b) *If* $rank(A) = m$ *and* $CA = 0$ *where* $C$ *is* $k \times m$*, show that* $C = 0$. 4 marks

4. *If* $A$ *is an* $m \times n$ *matrix, with columns* $C_1, C_2, \cdots, C_n$*. If* $\{C_1, C_2, \cdots, C_n\}$ *is an orthogonal set in* $\mathbb{R}^m$*, show that* $A^T A$ *is a diagonal* $n \times n$ *matrix, and describe its diagonal entries.* 8 marks

5. *If* $A$ *is an* $m \times n$ *matrix, show that* $\lambda \geq 0$ *for every eigenvalue* $\lambda$ *of* $A^T A$. [Hint: $\|X\|^2 = X^T X$ for every column $X$ in $\mathbb{R}^n$.] 8 marks

Total: 40 marks