ASSIGNMENT 3.

MATH 311 **FALL 2010**

1. Let $A = \begin{bmatrix} 1 & -1 & -1 & 2 & 2 & 3 \\ -2 & 2 & 3 & -6 & -3 & -11 \\ -1 & 1 & 2 & -4 & -3 & -8 \\ 0 & 0 & 1 & -2 & 1 & -5 \end{bmatrix}$.

   (1) Find bases of $row(A)$ and of $col(A)$, and so determine their dimensions. 4 marks

   (2) Find bases of $null(A)$ and of $im(A)$, and so determine their dimensions. 4 marks

2. *Let* $\mathcal{B} = \{(2,1,1,-1),\ (1,-3,1,0),\ (-1,3,10,11),\ (1,0,-1,1)\}$ *in* $\mathbb{R}^4$.

   (1) *Show that* $\mathcal{B}$ *is a basis of* $\mathbb{R}^4$. 4 marks

   (2) *Express* $X = (a,b,c,d)$ *as a linear combination of the vectors in* $\mathcal{B}$. 4 marks

3. *Let* A *be an* $m \times n$ *matrix with* $rank(A) = n$. *If the columns of* A *are* $C_1, C_2, \ldots, C_n$, *show that* $\mathcal{B} = \{A^T C_1,\ A^T C_2, \ldots, A^T C_n\}$ *is a basis of* $\mathbb{R}^n$. 8 marks

4. *Let* A *and* B *denote* $n \times n$ *matrices. Write* $A \sim B$ *to mean that* A *is similar to* B.

   (1) *If* A *is invertible, show that* $AB \sim BA$. 2 marks

   (2) *Let* $A = rI_n$ *where* $r \in \mathbb{R}$. *If* $B \sim A$, *show that* $B = A$. 3 marks

   (3) *If* $A \sim B$ *and* $A^3 = A$, *show that* $B^3 = B$. 3 marks

5. *Let* A *be an* $n \times n$ *diagonalizable matrix with eigenvalues* $\lambda_1,\ \lambda_2, \ldots, \lambda_n$ (*including multiplicities*).

   (1) *Show that* $det(A) = \lambda_1 \lambda_2 \ldots \lambda_n$. 4 marks

   (2) *Show that* $tr(A) = \lambda_1 + \lambda_2 + \ldots + \lambda_n$ (*where* $tr(A)$ *denotes the trace of* A). 4 marks

Total: 40 marks