ASSIGNMENT 5.

MATH 311 **FALL 2010**

1. #13 §8.1. *If U is a subspace of $\mathbb{R}^n$ show that $U^{\perp\perp} = U$ where $U^{\perp\perp} = (U^{\perp})^{\perp}$.* [Hint: Show that $U \subseteq U^{\perp\perp}$ and use Theorem 4 §8.1 twice.] 8 marks

2. *If A is an $m \times n$ matrix, show that $null(A) = [row(A)]^{\perp}$.* 8 marks

3. #13 §8.1. *Assume that A and B are orthogonally similar (that is $P^T AP = B$ for some orthogonal matrix P).*

   (a) *If A and B are invertible, show that A^{-1} and B^{-1} are orthogonally similar.* 3 marks

   (b) *Show that A^2 and B^2 are orthogonally similar.* 2 marks

   (c) *Show that if A is symmetric, so also is B.* 3 marks

4. #5 §8.3. *If A and B are positive definite, show that $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ is positive definite.* 8 marks

5. (a) #9 §8.6. *Show that $\langle ZX, Y \rangle = \langle X, Z^*Y \rangle$ for all $n \times n$ matrices Z and all X and Y in $\mathbb{C}^n$.*

   #15 §8.6. *Let U be a unitary matrix. Show that*

   (b) $\|UX\| = \|X\|$ *for all X in $\mathbb{C}^n$.*

   (c) $|\lambda| = 1$ *for every eigenvalue λ of U.* 8 marks

Total: 40 marks