ASSIGNMENT 1 SOLUTIONS.

MATH 311 **FALL 2010**

1. *If* $\mathbb{R}^n = span\{X_1, X_2, \cdots, X_k\}$, *and* $A \neq 0$ *is* $m \times n$, *show that* $AX_i \neq 0$ *for some* i. 7 marks

2. *Let* A *denote an* $m \times n$ *matrix. Show that* $dim[null(A)] = dim[null(AV)]$ *for any invertible* $n \times n$ *matrix* V. [*Hint*: *If* $AX = 0$ *then* $AV(V^{-1}X) = 0$.] 7 marks

3. *Show that* $\{x^2 + 2,\ 3x - x^3,\ 4 + x + x^3, x\}$ *spans* $\mathbf{P}_3$. 6 marks

4. *Let* $U = span\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \cdots, \mathbf{v}_k\}$ *in a vector space* V. *If* $\mathbf{w} = a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 + \cdots + a_k\mathbf{v}_k$ *where* $a_2 \neq 0$, *show that* $U = span\{\mathbf{v}_1, \mathbf{w}, \mathbf{v}_3, \cdots, \mathbf{v}_k\}$. 7 marks

5. *Among all independent sets in a vector space* V, *let* $B = \{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k\}$ *be an independent set where* k *is as large as possible. Show that* B *is a basis of* V. 7 marks

6. *Find the dimension of the subspace* U *of* $\mathbb{R}^5$ if $U = \left\{ \begin{bmatrix} a-b \\ c+2b \\ 3b+c \\ c-a \\ 3a-2b+5c \end{bmatrix} \mid a, b, c \text{ in } \mathbb{R} \right\}$. 6 marks

Total: 40 marks