UNIVERSITY OF CALGARY

Faculty of Science
Dept. of Mathematics & Statistics

Honours Linear Algebra
MATH 313 - Winter 2011
Homework #1

Due Monday January 24, 2011
(Not all questions will be marked)

Let $s1, s2, s3, s4, s5, s6, s7, s8$ denote the full 8 digits of your birth (in the form $mmddyyyy$).

1. (a) Give an example of a vector space V over the field $\mathbb{Q}$, but try to choose it in such a way that no other student has used the same V as their answer to this question.
   (b) Give an example of an $(s2+s4)$-dimensional vector space V over the field $\mathbb{C}$, and write down a basis for V.
   (c) Give a list of $s2+3$ vectors which span a vector space (of your choosing) but do not form a basis for this vector space.

2. (a) Show that the collection $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ is a basis for the vector space $V = \mathbb{R}^2$ over the field $\mathbb{R}$, by first showing that the two vectors are linearly independent, and secondly showing that they are spanning.
   (b) Decide whether the collection $B' = \left\{ \begin{bmatrix} s1 \\ s2 \end{bmatrix}, \begin{bmatrix} s3 \\ s4 \end{bmatrix} \right\}$ is a linearly independent subset of $\mathbb{R}^2$. Is B' spanning $\mathbb{R}^2$? Is B' a basis for $\mathbb{R}^2$? Justify your answers.
   (c) For each vector in B', determine its coordinate representation with respect to the basis B
   (d) If your list B' is a basis, find the transition matrix $P = P_{B;B'}$ from B to B' (hint: use your answer to (c)). If your list B' is not a basis, then justify why exactly it isn't.

3. Using MAPLE (or other computational tool) if you wish, repeat the previous question using the collections $B = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

   and

$$B' = \left\{ \begin{bmatrix} s1 \\ s2 \\ s3 \\ s4 \end{bmatrix}, \begin{bmatrix} s2 \\ s3 \\ s4 \\ s5 \end{bmatrix}, \begin{bmatrix} s3 \\ s4 \\ s5 \\ s6 \end{bmatrix}, \begin{bmatrix} s4 \\ s5 \\ s6 \\ s7 \end{bmatrix} \right\}.$$

4. Consider the set $\mathbb{P}_n$ of all polynomials of degree at most n. That is

$$\mathbb{P}_n = \{\sum_{i=0}^{n} a_i x^i : a_i \in \mathbb{R}\}$$

Define addition and scalar multiplication in the usual manner as follows:

$$\sum_{i=0}^{n} a_i x^i + \sum_{i=0}^{n} b_i x^i = \sum_{i=0}^{n} (a_i + b_i) x^i$$
$$k \cdot \sum_{i=0}^{n} a_i x^i = \sum_{i=0}^{n} k a_i x^i$$

(a) Convince yourself that $\mathbb{P}_n$ is a vector space.

(b) Find a basis of $\mathbb{P}_n$, show it is a basis, and hence find its dimension.

5. State whether the following are True or False, giving reasons.

6. (a) The set $U = \left\{ \begin{bmatrix} t \\ t^2 \end{bmatrix} : t \in \mathbb{R} \right\}$ is a vector subspace of $\mathbb{R}^2$.

   (b) The set $U = \left\{ \begin{bmatrix} 0 \\ s \\ s+t \end{bmatrix} : s, t \in \mathbb{R} \right\}$ is a vector subspace of $\mathbb{R}^2$.

7. Prove Proposition 1.1 in detail.

8. Let V be a finite-dimensional vector space, and let U be a subspace of V. Use the results of Chapter 1 to show that $dim(U) \leq dim(V)$.