UNIVERSITY OF CALGARY

Faculty of Science
Dept. of Mathematics & Statistics

Honours Linear Algebra
MATH 313 - Winter 2011
Homework #2

Due Monday February 7, 2011
(Not all questions will be marked)

1. Hand in the Homework #1 problems you may not have handed before.

2. (a) What are the possible dimensions of subspaces of $\mathbb{R}^3$? Explain.
   (b) If U and V are two subspaces of $\mathbb{R}^3$ of dimension 2, what are the possible dimensions for $U \cap V$? Explain.
   (c) Verify by examples that these possibilities do exist.

3. (a) Let A be an $n \times n$ matrix. Show $null(A) = \{X : AX = 0\}$ is a subspace of $\mathbb{R}^n$.
   (b) Let U be any subspace of $\mathbb{R}^2$. Show that $U = null(A)$ for some matrix A.
   (c) Let U be any subspace of $\mathbb{R}^3$. Show that $U = null(A)$ for some matrix A.
   (d) Let U be any subspace of $\mathbb{R}^n$. Show that $U = null(A)$ for some matrix A.

4. Prove (in your own words and proof) that if U and V and subspaces of a finite dimensional space W, then

$$dim(U \cap V) + dim(U + V) = dim(U) + dim(V)$$

5. Let D be a determinant on $n \times n$ matrices and let A be non invertible. Write a detailed, complete and clear proof that $D(A) = 0$ (i.e. expanding on the P. 24 argument).

6. Let V be a finite dimensional vector space. Argue whether the following pseudo algorithm necessarily terminates and produces a basis.

   Let $B = \emptyset$.
   While B does not span V: Find $v \in V$ not in the span of B, and let $B = B \cup \{v\}$.

7. Bonus Project (can be handed in at any time)

   (a) Let $U = \langle u_1, \ldots, u_k \rangle$ and $V = \langle v_1, \ldots, v_k \rangle$ be two subspaces of $\mathbb{R}^n$. Explain how one would find a basis of $U \cap V$.
   (b) Write a pseudo algorithm (meaning something in your own words that sounds like a computer program) such that on input the vectors of $\mathbb{R}^n$ $\langle u_1, \ldots, u_k \rangle$ spanning U and $\langle v_1, \ldots, v_k \rangle$ spanning V, the algorithm outputs a basis of $U \cap V$.
   (c) Write a working algorithm (in your favourite computing package) such that on input the vectors of $\mathbb{R}^n$ $\langle u_1, \ldots, u_k \rangle$ spanning U and $\langle v_1, \ldots, v_k \rangle$ spanning V, the algorithm outputs a basis of $U \cap V$.