UNIVERSITY OF CALGARY

FACULTY OF SCIENCE
Department of Mathematics and Statistics

Mathematics 315 Algebra I

(see Course Descriptions under the year applicable: http://www.ucalgary.ca/pubs/calendar/)

Syllabus

Topics	**Number of hours**
Review: primes, divisibility, the Euclidean algorithm; the integers modulo n; modular arithmetic	6
Group axioms; cyclic and dihedral groups; matrix groups; permutations and the symmetric group	3
Subgroups; cosets and Lagrange's theorem; normal subgroups	3
Homormorphisms and kernels; quotient groups; the isomorphism theorems; examples	3
Group actions and Cayley's theorem; orbit counting formula; combinatorial applications; free groups and presentations (time permitting)	3
Ring axioms; division rings and fields; Z/nZ is a field if and only if n is prime; examples	3
The ring of polynomials over a field; greatest common divisor and the Euclidean algorithm; irreducible polynomials	3
Integral domains, unique factorization domains, principal ideal domains and Euclidean domains; ideals; Z and F[x] are Euclidean domains	5
Homomorphisms and kernels; quotients, first isomorphism theorem	3
Adjoining the root of an irreducible polynomial; construction of finite fields	4
TOTAL HOURS	**36**

Mathematics 315 Algebra I
Course Outcomes

The main objective of this course is to provide students with a thorough understanding of the fundamentals of abstract algebra and the techniques used to construct, analyze and manipulate algebraic objects. Students will acquire a solid understanding of sets and maps, properties of integers and polynomials, groups and rings. They will develop the ability to apply proof techniques as well as algebraic and algorithmic manipulations to algebraic objects and structures. In particular, a student who successfully completes this course will be able to:

1. Apply basic proof techniques, including direct proofs, proofs by contraction and mathematical induction to rigorously prove results about abstract and concrete algebraic objects

2. Carry out manipulations on concrete algebraic objects, including sets, maps, permutations, integers and polynomials

3. Apply algorithmic techniques to integers and polynomials, including
Euclidean division, greatest common divisor computation, modular arithmetic and solving systems of linear congruences

4. Build new structures by applying algebraic constructions, including products and quotients, to basic algebraic objects

5. Describe and prove fundamental facts about and properties of groups, rings and homomorphisms

6. Demonstrate proficiency in applying structural properties of groups, rings and homomorphisms to solve concrete problems, including applications arising in other areas of mathematics such as geometry and number theory

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