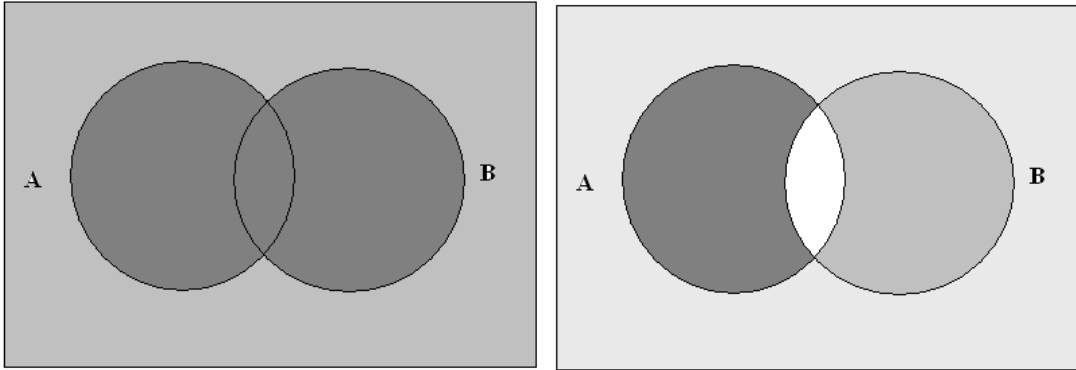


Chapter 2: Probability

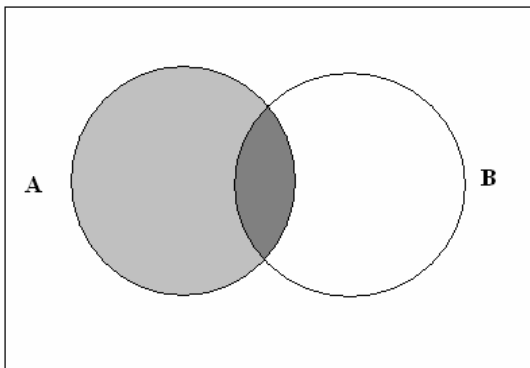
2.1 $A = \{FF\}, B = \{MM\}, C = \{MF, FM, MM\}$. Then, $A \cap B = \emptyset, B \cap C = \{MM\}, C \cap \bar{B} = \{MF, FM\}, A \cup B = \{FF, MM\}, A \cup C = S, B \cup C = C$.

2.2 a. $A \cap B$ b. $A \cup B$ c. $\overline{A \cup B}$ d. $(A \cap \bar{B}) \cup (\bar{A} \cap B)$

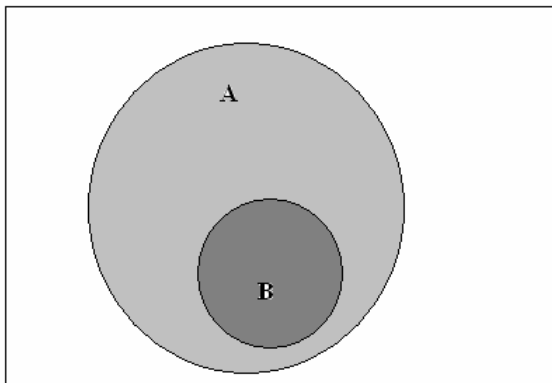
2.3



2.4 a.



b.



- 2.5**
- a. $(A \cap B) \cup (A \cap \bar{B}) = A \cap (B \cup \bar{B}) = A \cap S = A$.
- b. $B \cup (A \cap \bar{B}) = (B \cap A) \cup (B \cap \bar{B}) = (B \cap A) = A$.
- c. $(A \cap B) \cap (A \cap \bar{B}) = A \cap (B \cap \bar{B}) = \emptyset$. The result follows from part a.
- d. $B \cap (A \cap \bar{B}) = A \cap (B \cap \bar{B}) = \emptyset$. The result follows from part b.
- 2.6**
- $A = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (1,6), (2,6), (3,6), (4,6), (5,6), (6,6)\}$
- $\bar{C} = \{(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)\}$
- $A \cap B = \{(2,2), (4,2), (6,2), (2,4), (4,4), (6,4), (2,6), (4,6), (6,6)\}$
- $A \cap \bar{B} = \{(1,2), (3,2), (5,2), (1,4), (3,4), (5,4), (1,6), (3,6), (5,6)\}$
- $\bar{A} \cup B = \text{everything but } \{(1,2), (1,4), (1,6), (3,2), (3,4), (3,6), (5,2), (5,4), (5,6)\}$
- $\bar{A} \cap C = \bar{A}$
- 2.7**
- $A = \{\text{two males}\} = \{(M_1, M_2), (M_1, M_3), (M_2, M_3)\}$
- $B = \{\text{at least one female}\} = \{(M_1, W_1), (M_2, W_1), (M_3, W_1), (M_1, W_2), (M_2, W_2), (M_3, W_2), \{W_1, W_2\}\}$
- $\bar{B} = \{\text{no females}\} = A \quad A \cup B = S \quad A \cap B = \emptyset \quad A \cap \bar{B} = A$
- 2.8**
- a. $36 + 6 = 42$ b. 33 c. 18
- 2.9**
- $S = \{A+, B+, AB+, O+, A-, B-, AB-, O-\}$
- 2.10**
- a. $S = \{A, B, AB, O\}$
- b. $P(\{A\}) = 0.41, P(\{B\}) = 0.10, P(\{AB\}) = 0.04, P(\{O\}) = 0.45$.
- c. $P(\{A\} \text{ or } \{B\}) = P(\{A\}) + P(\{B\}) = 0.51$, since the events are mutually exclusive.
- 2.12**
- a. Let $L = \{\text{left turn}\}, R = \{\text{right turn}\}, C = \{\text{continues straight}\}$.
- b. $P(\text{vehicle turns}) = P(L) + P(R) = 1/3 + 1/3 = 2/3$.
- 2.14**
- a. $P(\text{needs glasses}) = .44 + .14 = .48$
- b. $P(\text{needs glasses but doesn't use them}) = .14$
- c. $P(\text{uses glasses}) = .44 + .02 = .46$
- 2.17**
- Let $B = \text{bushing defect}, SH = \text{shaft defect}$.
- a. $P(B) = .06 + .02 = .08$
- b. $P(B \text{ or } SH) = .06 + .08 + .02 = .16$
- c. $P(\text{exactly one defect}) = .06 + .08 = .14$
- d. $P(\text{neither defect}) = 1 - P(B \text{ or } SH) = 1 - .16 = .84$
- 2.18**
- a. $S = \{HH, TH, HT, TT\}$
- b. if the coin is fair, all events have probability .25.
- c. $A = \{HT, TH\}, B = \{HT, TH, HH\}$
- d. $P(A) = .5, P(B) = .75, P(A \cap B) = P(A) = .5, P(A \cup B) = P(B) = .75, P(\bar{A} \cup B) = 1$.

- 2.26** a. Let N_1, N_2 denote the empty cans and W_1, W_2 denote the cans filled with water. Thus, $S = \{N_1N_2, N_1W_2, N_2W_2, N_1W_1, N_2W_1, W_1W_2\}$
b. If this a merely a guess, the events are equally likely. So, $P(W_1W_2) = 1/6$.
- 2.27** a. $S = \{CC, CR, CL, RC, RR, RL, LC, LR, LL\}$
b. $5/9$
c. $5/9$
- 2.28a.** Denote the four candidates as A_1, A_2, A_3 , and M . Since order is not important, the outcomes are $\{A_1A_2, A_1A_3, A_1M, A_2A_3, A_2M, A_3M\}$.
b. assuming equally likely outcomes, all have probability $1/6$.
c. $P(\text{minority hired}) = P(A_1M) + P(A_2M) + P(A_3M) = .5$
- 2.32** a. Let "1" represent a customer seeking style 1, and "2" represent a customer seeking style 2. The sample space consists of the following 16 four-tuples:
1111, 1112, 1121, 1211, 2111, 1122, 1212, 2112, 1221, 2121,
2211, 2221, 2212, 2122, 1222, 2222
b. If the styles are equally in demand, the ordering should be equally likely. So, the probability is $1/16$.
c. $P(A) = P(1111) + P(2222) = 2/16$.
- 2.34** a. Three patients enter the hospital and randomly choose stations 1, 2, or 3 for service. Then, the sample space S contains the following 27 three-tuples:
111, 112, 113, 121, 122, 123, 131, 132, 133, 211, 212, 213, 221, 222, 223,
231, 232, 233, 311, 312, 313, 321, 322, 323, 331, 332, 333
b. $A = \{123, 132, 213, 231, 312, 321\}$
c. If the stations are selected at random, each sample point is equally likely. $P(A) = 6/27$.
- 2.35** The total number of flights is $6(7) = 42$.
- 2.36** There are $3! = 6$ orderings.
- 2.37** a. There are $6! = 720$ possible itineraries.
b. In the 720 orderings, exactly 360 have Denver before San Francisco and 360 have San Francisco before Denver. So, the probability is $.5$.
- 2.38** By the multiplication rule, $4(3)(4)(5) = 240$.
- 2.39** a. By the multiplication rule, there are $6(6) = 36$ possible roles.
b. Define the event $A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$. Then, $P(A) = 6/36$.
- 2.40** a. By the multiplication rule, the dealer must stock $5(4)(2) = 40$ autos.
b. To have each of these in every one of the eight colors, he must stock $8*40 = 320$ autos.

2.41 If the first digit cannot be zero, there are 9 possible values. For the remaining six, there are 10 possible values. Thus, the total number is $9(10)(10)(10)(10)(10) = 9 \cdot 10^6$.

2.42 There are three *different* positions to fill using ten engineers. Then, there are $P_3^{10} = 10!/3! = 10 \cdot 9 \cdot 8 = 720$ different ways to fill the positions.

2.51 There are $\binom{50}{3} = 19,600$ ways to choose the 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3} = 4$ ways for the organizers to win all of the prizes. The probability is $4/19600$.

b. There are $\binom{4}{2} \binom{46}{1} = 276$ ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $276/19600$.

c. $\binom{4}{1} \binom{46}{2} = 4140$. The probability is $4140/19600$.

d. $\binom{46}{3} = 15,180$. The probability is $15180/19600$.

2.53 a. In choosing three of the five firms, order is important. So $P_3^5 = 60$ sample points.

b. If F_3 is awarded a contract, there are $P_2^4 = 12$ ways the other contracts can be assigned. Since there are 3 possible contracts, there are $3(12) = 36$ total number of ways to award F_3 a contract. So, the probability is $36/60 = 0.6$.

2.56 The student can solve all of the problems if the teacher selects 5 of the 6 problems that the student can do. The probability is $\binom{6}{5} / \binom{10}{5} = 0.0238$.

2.57 There are $\binom{52}{2} = 1326$ ways to draw two cards from the deck. The probability is $4 \cdot 12 / 1326 = 0.0362$.

2.58 There are $\binom{52}{5} = 2,598,960$ ways to draw five cards from the deck.

a. There are $\binom{4}{3} \binom{4}{2} = 24$ ways to draw three Aces and two Kings. So, the probability is $24/2598960$.

b. There are $13(12) = 156$ types of “full house” hands. From part a. above there are 24 different ways each type of full house hand can be made. So, the probability is $156 \cdot 24 / 2598960 = 0.00144$.

2.59 There are $\binom{52}{5} = 2,598,960$ ways to draw five cards from the deck.

a. $\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1}\binom{4}{1} = 4^5 = 1024$. So, the probability is $1024 / 2598960 = 0.000394$.

2.64 $6! \left(\frac{1}{6}\right)^6 = 5/324$.

2.65 $5! \left(\frac{2}{6}\right)^6 \left(\frac{1}{6}\right)^4 = 5/162$.

2.66 a. After assigning an ethnic group member to each type of job, there are 16 laborers remaining for the other jobs. Let n_a be the number of ways that one ethnic group can be assigned to each type of job. Then:

$$n_a = \binom{4}{1\ 1\ 1\ 1} \binom{16}{5\ 3\ 4\ 4}. \text{ The probability is } n_a/N = 0.1238.$$

b. It doesn't matter how the ethnic group members are assigned to jobs type 1, 2, and 3. Let n_a be the number of ways that no ethnic member gets assigned to a type 4 job. Then:

$$n_a = \binom{4}{0} \binom{16}{5}. \text{ The probability is } \binom{4}{0} \binom{16}{5} / \binom{20}{5} = 0.2817.$$

2.71 a. $P(A|B) = .1/.3 = 1/3$.

b. $P(B|A) = .1/.5 = 1/5$.

c. $P(A|A \cup B) = .5/ (.5 + .3 - .1) = 5/7$

d. $P(A|A \cap B) = 1$, since A has occurred.

e. $P(A \cap B | A \cup B) = .1 / (.5 + .3 - .1) = 1/7$.

2.72 Note that $P(A) = 0.6$ and $P(A|M) = .24/.4 = 0.6$. So, A and M are independent. Similarly, $P(\bar{A} | F) = .24/.6 = 0.4 = P(\bar{A})$, so \bar{A} and F are independent.

2.75. Given the first two cards drawn are spades, there are 11 spades left in the deck. Thus,

the probability is $\frac{\binom{11}{3}}{\binom{50}{3}} = 0.0084$. Note: this is also equal to $P(S_3S_4S_5|S_1S_2)$.

b. Given the first three cards drawn are spades, there are 10 spades left in the deck. Thus,

the probability is $\frac{\binom{10}{2}}{\binom{49}{2}} = 0.0383$. Note: this is also equal to $P(S_4S_5|S_1S_2S_3)$.

c. Given the first four cards drawn are spades, there are 9 spades left in the deck. Thus,

the probability is $\frac{\binom{9}{1}}{\binom{48}{1}} = 0.1875$. Note: this is also equal to $P(S_5|S_1S_2S_3S_4)$

2.76 Define the events: U : job is unsatisfactory A : plumber A does the job

a. $P(U|A) = P(A \cap U)/P(A) = P(A|U)P(U)/P(A) = .5 \cdot .1/.4 = 0.125$

b. From part a. above, $1 - P(U|A) = 0.875$.

2.78 **1.** Assume $P(A|B) = P(A)$. Then:

$P(A \cap B) = P(A|B)P(B) = P(A)P(B)$. $P(B|A) = P(B \cap A)/P(A) = P(A)P(B)/P(A) = P(B)$.

2. Assume $P(B|A) = P(B)$. Then:

$P(A \cap B) = P(B|A)P(A) = P(B)P(A)$. $P(A|B) = P(A \cap B)/P(B) = P(A)P(B)/P(B) = P(A)$.

3. Assume $P(A \cap B) = P(B)P(A)$. The results follow from above.

2.80 If $A \subset B$, $P(A \cap B) = P(A) \neq P(A)P(B)$, unless $B = S$ (in which case $P(B) = 1$).

2.83 $P(A|A \cup B) = P(A)/P(A \cup B) = \frac{P(A)}{P(A) + P(B)}$, since A and B are M.E. events.

2.85 $P(A|\bar{B}) = P(A \cap \bar{B})/P(\bar{B}) = \frac{P(\bar{B}|A)P(A)}{P(\bar{B})} = \frac{[1 - P(B|A)]P(A)}{P(\bar{B})} = \frac{[1 - P(B)]P(A)}{P(\bar{B})} =$

$\frac{P(\bar{B})P(A)}{P(\bar{B})} = P(A)$. So, A and \bar{B} are independent.

$$P(\bar{B} | \bar{A}) = P(\bar{B} \cap \bar{A}) / P(\bar{A}) = \frac{P(\bar{A} | \bar{B})P(\bar{B})}{P(\bar{A})} = \frac{[1 - P(A | \bar{B})]P(\bar{B})}{P(\bar{A})}. \text{ From the above,}$$

$$A \text{ and } \bar{B} \text{ are independent. So } P(\bar{B} | \bar{A}) = \frac{[1 - P(A)]P(\bar{B})}{P(\bar{A})} = \frac{P(\bar{A})P(\bar{B})}{P(\bar{A})} = P(\bar{B}). \text{ So,}$$

\bar{A} and \bar{B} are independent

2.90 a. $(1/50)(1/50) = 0.0004.$

b. $P(\text{at least one injury}) = 1 - P(\text{no injuries in 50 jumps}) = 1 - (49/50)^{50} = 0.636.$ Your friend is not correct.

2.93 Let H denote a hit and let M denote a miss. Then, she wins the game in three trials with the events HHH, HHM, and MHH. If she begins with her right hand, the probability she wins the game, assuming independence, is $(.7)(.4)(.7) + (.7)(.4)(.3) + (.3)(.4)(.7) = 0.364.$

2.94 Define the events A : device A detects smoke B : device B detects smoke

a. $P(A \cup B) = .95 + .90 - .88 = 0.97.$

b. $P(\text{smoke is undetected}) = 1 - P(A \cup B) = 1 - 0.97 = 0.03.$

2.95 Part a is found using the Addition Rule. Parts b and c use DeMorgan's Laws.

a. $0.2 + 0.3 - 0.4 = 0.1$

b. $1 - 0.1 = 0.9$

c. $1 - 0.4 = 0.6$

d. $P(\bar{A} | B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{.3 - .1}{.3} = 2/3.$

2.97 a. $P(\text{current flows}) = 1 - P(\text{all three relays are open}) = 1 - (.1)^3 = 0.999.$

b. Let A be the event that current flows and B be the event that relay 1 closed properly. Then, $P(B|A) = P(B \cap A) / P(A) = P(B) / P(A) = .9 / .999 = 0.9009.$ Note that $B \subset A.$

2.101 Let A be the event the item gets past the first inspector and B the event it gets past the second inspector. Then, $P(A) = 0.1$ and $P(B|A) = 0.5.$ Then $P(A \cap B) = .1(.5) = 0.05.$

2.102 Define the events: I : disease I us contracted II : disease II is contracted. Then, $P(I) = 0.1, P(II) = 0.15,$ and $P(I \cap II) = 0.03.$

a. $P(I \cup II) = .1 + .15 - .03 = 0.22$

b. $P(I \cap II | I \cup II) = .03 / .22 = 3/22.$

2.109 This is similar to Ex. 2.106. Apply Ex. 2.108: $0.95 \leq 1 - P(\bar{A}) - P(\bar{B}) - P(\bar{C}) \leq P(A \cap B \cap C).$ Since the events have the same probability, $0.95 \leq 1 - 3P(\bar{A}).$ Thus, $P(A) \geq 0.9833.$

2.110 Define the events:

I : item is from line I II : item is from line II N : item is not defective

Then, $P(N) = P(N \cap (I \cup II)) = P(N \cap I) + P(N \cap II) = .92(.4) + .90(.6) = 0.908$.

2.112 a. $P(\text{aircraft undetected}) = P(\text{all three fail to detect}) = (.02)(.02)(.02) = (.02)^3$.

b. $P(\text{all three detect aircraft}) = (.98)^3$.

2.114 Let $T = \{\text{detects truth}\}$ and $L = \{\text{detects lie}\}$. The sample space is TT, TL, LT, LL . Since one suspect is guilty, assume the guilty suspect is questioned first:

a. $P(LL) = .95(.10) = 0.095$ **b.** $P(LT) = .95(.9) = 0.885$

b. $P(TL) = .05(.10) = 0.005$ **d.** $1 - (.05)(.90) = 0.955$

2.115 By independence, $(.75)(.75)(.75)(.75) = (.75)^4$.

2.116 By the complement rule, $P(\text{system works}) = 1 - P(\text{system fails}) = 1 - (.01)^3$.

2.119 a. Define the events: A : obtain a sum of 3 B : do not obtain a sum of 3 or 7
Since there are 36 possible rolls, $P(A) = 2/36$ and $P(B) = 28/36$. Obtaining a sum of 3 before a sum of 7 can happen on the 1st roll, the 2nd roll, the 3rd roll, etc. Using the events above, we can write these as $A, BA, BBA, BBBA$, etc. The probability of obtaining a sum of 3 before a sum of 7 is given by $P(A) + P(B)P(A) + [P(B)]^2P(A) + [P(B)]^3P(A) + \dots$. (Here, we are using the fact that the rolls are independent.) This is an infinite sum, and it follows as a geometric series. Thus, $2/36 + (28/36)(2/36) + (28/36)^2(2/36) + \dots = 1/4$.

b. Similar to part a. Define C : obtain a sum of 4 D : do not obtain a sum of 4 or 7
Then, $P(C) = 3/36$ and $P(D) = 27/36$. The probability of obtaining a 4 before a 7 is $1/3$.

2.120 Denote the events G : good refrigerator D : defective refrigerator

a. If the last defective refrigerator is found on the 4th test, this means the first defective refrigerator was found on the 1st, 2nd, or 3rd test. So, the possibilities are $DGDD, GDGD$, and $GGDD$. So, the probability is $(\frac{2}{6})(\frac{4}{5})(\frac{3}{4})\frac{1}{3}$. The probabilities associated with the other two events are identical to the first. So, the desired probability is $3(\frac{2}{6})(\frac{4}{5})(\frac{3}{4})\frac{1}{3} = \frac{1}{5}$.

b. Here, the second defective refrigerator must be found on the 2nd, 3rd, or 4th test.

Define: A_1 : second defective found on 2nd test

A_2 : second defective found on 3rd test

A_3 : second defective found on 4th test

Clearly, $P(A_1) = (\frac{2}{6})(\frac{1}{5}) = \frac{1}{15}$. Also, $P(A_3) = \frac{1}{5}$ from part a. Note that $A_2 = \{DGD, GDD\}$.

Thus, $P(A_2) = 2(\frac{2}{6})(\frac{4}{5})(\frac{1}{4}) = \frac{2}{15}$. So, $P(A_1) + P(A_2) + P(A_3) = 2/5$.

c. Define: B_1 : second defective found on 3rd test

B_2 : second defective found on 4th test

Clearly, $P(B_1) = 1/4$ and $P(B_2) = (3/4)(1/3) = 1/4$. So, $P(B_1) + P(B_2) = 1/2$.

2.124 Define the events for the voter: D : democrat R : republican F : favors issue

$$P(D|F) = \frac{P(F|D)P(D)}{P(F|D)P(D) + P(F|R)P(R)} = \frac{.7(.6)}{.7(.6) + .3(.4)} = 7/9$$

2.125 Define the events for the person: D : has the disease H : test indicates the disease

Thus, $P(H|D) = .9$, $P(\bar{H}|\bar{D}) = .9$, $P(D) = .01$, and $P(\bar{D}) = .99$. Thus,

$$P(D|H) = \frac{P(H|D)P(D)}{P(H|D)P(D) + P(H|\bar{D})P(\bar{D})} = 1/12.$$

2.129 Define the events: P : positive response M : male respondent F : female respondent

$P(P|F) = .7$, $P(P|M) = .4$, $P(M) = .25$. Using Bayes' rule,

$$P(M|\bar{P}) = \frac{P(\bar{P}|M)P(M)}{P(\bar{P}|M)P(M) + P(\bar{P}|F)P(F)} = \frac{.6(.25)}{.6(.25) + .3(.75)} = 0.4.$$

2.130 Define the events: C : contract lung cancer S : worked in a shipyard

Thus, $P(S|C) = .22$, $P(S|\bar{C}) = .14$, and $P(C) = .0004$. Using Bayes' rule,

$$P(C|S) = \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|\bar{C})P(\bar{C})} = \frac{.22(.0004)}{.22(.0004) + .14(.9996)} = 0.0006.$$

2.132 For $i = 1, 2, 3$, let F_i represent the event that the plane is found in region i and N_i be the complement. Also R_i is the event the plane is in region i . Then $P(F_i|R_i) = 1 - \alpha_i$ and $P(R_i) = 1/3$ for all i . Then,

$$\begin{aligned} \text{a. } P(R_1|N_1) &= \frac{P(N_1|R_1)P(R_1)}{P(N_1|R_1)P(R_1) + P(N_1|R_2)P(R_2) + P(N_1|R_3)P(R_3)} = \frac{\alpha_1 \frac{1}{3}}{\alpha_1 \frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \\ &= \frac{\alpha_1}{\alpha_1 + 2}. \end{aligned}$$

$$\text{b. Similarly, } P(R_2|N_1) = \frac{1}{\alpha_1 + 2} \text{ and } \text{c. } P(R_3|N_1) = \frac{1}{\alpha_1 + 2}.$$

2.133 Define the events: G : student guesses C : student is correct

$$P(\bar{G}|C) = \frac{P(C|\bar{G})P(\bar{G})}{P(C|\bar{G})P(\bar{G}) + P(C|G)P(G)} = \frac{1(.8)}{1(.8) + .25(.2)} = 0.9412.$$

2.134 Define F as "failure to learn. Then, $P(F|A) = .2$, $P(F|B) = .1$, $P(A) = .7$, $P(B) = .3$. By Bayes' rule, $P(A|F) = 14/17$.

2.135 Let M = major airline, P = private airline, C = commercial airline, B = travel for business

$$\text{a. } P(B) = P(B|M)P(M) + P(B|P)P(P) + P(B|C)P(C) = .6(.5) + .3(.6) + .1(.9) = 0.57.$$

$$\text{b. } P(B \cap P) = P(B|P)P(P) = .3(.6) = 0.18.$$

$$\text{c. } P(P|B) = P(B \cap P)/P(B) = .18/.57 = 0.3158.$$

d. $P(B|C) = 0.90$.

2.138. Define the events:

A : the player wins

B_i : a sum of i on first toss

C_k : obtain a sum of k before obtaining a 7

Now, $P(A) = \sum_{i=1}^{12} P(A \cap B_i)$. We have that $P(A \cap B_2) = P(A \cap B_3) = P(A \cap B_{12}) = 0$.

Also, $P(A \cap B_7) = P(B_7) = \frac{6}{36}$, $P(A \cap B_{11}) = P(B_{11}) = \frac{2}{36}$.

Now, $P(A \cap B_4) = P(C_4 \cap B_7) = P(C_4)P(B_7) = \frac{1}{3}(\frac{3}{36}) = \frac{3}{36}$ (using independence Ex. 119).

Similarly, $P(C_5) = P(C_9) = \frac{4}{10}$, $P(C_6) = P(C_8) = \frac{5}{11}$, and $P(C_{10}) = \frac{3}{9}$.

Thus, $P(A \cap B_5) = P(A \cap B_9) = \frac{2}{45}$, $P(A \cap B_6) = P(A \cap B_8) = \frac{25}{396}$, $P(A \cap B_{10}) = \frac{1}{36}$.

Putting all of this together, $P(A) = 0.493$.

2.136 From Ex. 1.112, $P(Y=0) = (.02)^3$ and $P(Y=3) = (.98)^3$. The event $Y=1$ are the events FDF , DFD , and FFD , each having probability $(.02)^2(.98)$. So, $P(Y=1) = 3(.02)^2(.98)$. Similarly, $P(Y=2) = 3(.02)(.98)^2$.

2.137 The total number of ways to select 3 from 6 refrigerators is $\binom{6}{3} = 20$. The total number of

ways to select y defectives and $3-y$ nondefectives is $\binom{2}{y} \binom{4}{3-y}$, $y = 0, 1, 2$. So,

$$P(Y=0) = \frac{\binom{2}{0} \binom{4}{3}}{20} = 4/20, P(Y=1) = 4/20, \text{ and } P(Y=2) = 12/20.$$

2.138 The events $Y=2$, $Y=3$, and $Y=4$ were found in Ex. 2.120 to have probabilities $1/15$, $2/15$, and $3/15$ (respectively). The event $Y=5$ can occur in four ways:

$DGGGD$ $GDGGD$ $GGDGD$ $GGGDD$

Each of these possibilities has probability $1/15$, so that $P(Y=5) = 4/15$. By the complement rule, $P(Y=6) = 5/15$.

2.139 Each position has probability $1/4$, so every ordering of two positions (from two spins) has

probability $1/16$. The values for Y are 2, 3. $P(Y=2) = \binom{4}{2} \frac{1}{16} = 3/4$. So, $P(Y=3) = 1/4$.

2.140 Since $P(B) = P(B \cap A) + P(B \cap \bar{A})$, $1 = \frac{P(B \cap A)}{P(B)} + \frac{P(B \cap \bar{A})}{P(B)} = P(A|B) + P(\bar{A}|B)$.

2.141a. $S = \{16 \text{ possibilities of drawing } 0 \text{ to } 4 \text{ of the sample points}\}$

b. $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 1 + 4 + 6 + 4 + 1 = 16 = 2^4.$

c. $A \cup B = \{E_1, E_2, E_3, E_4\}, A \cap B = \{E_2\}, \bar{A} \cap \bar{B} = \emptyset, \bar{A} \cup B = \{E_2, E_4\}.$

2.142 All 18 orderings are possible, so the total number of orderings is 18!

2.143 There are $\binom{52}{5}$ ways to draw 5 cards from the deck. For each suit, there are $\binom{13}{5}$ ways to select 5 cards. Since there are 4 suits, the probability is $4\binom{13}{5} / \binom{52}{5} = 0.00248.$

2.144 The gambler will have a full house if he is dealt {two kings} or {an ace and a king} (there are 47 cards remaining in the deck, two of which are aces and three are kings). The probabilities of these two events are $\binom{3}{2} / \binom{47}{2}$ and $\binom{3}{1}\binom{2}{1} / \binom{47}{2}$, respectively. So, the probability of a full house is $\binom{3}{2} / \binom{47}{2} + \binom{3}{1}\binom{2}{1} / \binom{47}{2} = 0.0083.$

2.145 Note that $\binom{12}{4} = 495$. P(each supplier has at least one component tested) is given by

$$\frac{\binom{3}{2}\binom{4}{1}\binom{5}{1} + \binom{3}{1}\binom{4}{2}\binom{5}{1} + \binom{3}{1}\binom{4}{1}\binom{5}{2}}{495} = 270/495 = 0.545.$$

2.146 Let A be the event that the person has symptom A and define B similarly. Then

a. $P(\overline{A \cup B}) = P(\bar{A} \cap \bar{B}) = 0.4$

b. $P(A \cup B) = 1 - P(\bar{A} \cap \bar{B}) = 0.6.$

c. $P(A \cap B | B) = P(A \cap B) / P(B) = .1 / .4 = 0.25$

2.147 $P(Y = 0) = 0.4, P(Y = 1) = 0.2 + 0.3 = 0.5, P(Y = 2) = 0.1.$

2.148 The probability that team A wins in 5 games is $p^4(1 - p)$ and the probability that team B wins in 5 games is $p(1 - p)^4$. Since there are 4 ways that each team can win in 5 games, the probability is $4[p^4(1 - p) + p(1 - p)^4].$

2.149 Let R denote the event that the specimen turns red and N denote the event that the specimen contains nitrates.

a. $P(R) = P(R|N)P(N) + P(R|\bar{N})P(\bar{N}) = .95(.3) + .1(.7) = 0.355.$

b. Using Bayes' rule, $P(N|R) = .95(.3)/.355 = 0.803.$

2.150 Using Bayes' rule,

$$P(I_1|H) = \frac{P(H|I_1)P(I_1)}{P(H|I_1)P(I_1) + P(H|I_2)P(I_2) + P(H|I_3)P(I_3)} = 0.313.$$

2.151 Let Y = the number of pairs chosen. Then, the possible values are 0, 1, and 2.

a. There are $\binom{10}{4} = 210$ ways to choose 4 socks from 10 and there are $\binom{5}{4} 2^4 = 80$ ways to pick 4 non-matching socks. So, $P(Y = 0) = 80/210.$

b. Generalizing the above, the probability is $\binom{n}{2r} 2^{2r} / \binom{2n}{2r}.$

2.152a. $P(A) = .25 + .1 + .05 + .1 = .5$

b. $P(A \cap B) = .1 + .05 = 0.15.$

c. 0.10

d. Using the result from Ex. 2.80, $\frac{.25 + .25 - .15}{.4} = 0.875.$

2.153a. i. $1 - 5686/97900 = 0.942$ ii. $(97900 - 43354)/97900 = 0.557$
 ii. $10560/14113 = 0.748$ iv. $(646+375+568)/11533 = 0.138$

b. If the US population in 2002 was known, this could be used to divide into the total number of deaths in 2002 to give a probability.

2.154 Let D denote death due to lung cancer and S denote being a smoker. Thus:

$$P(D) = P(D|S)P(S) + P(D|\bar{S})P(\bar{S}) = 10P(D|\bar{S})(.2) + P(D|S)(.8) = 0.006. \text{ Thus, } P(D|S) = 0.021.$$

2.155 Let W denote the event that the first ball is white and B denote the event that the second ball is black. Then:

$$P(W|B) = \frac{P(B|W)P(W)}{P(B|W)P(W) + P(B|\bar{W})P(\bar{W})} = \frac{\frac{b}{w+b+n} \left(\frac{w}{w+b} \right)}{\frac{b}{w+b+n} \left(\frac{w}{w+b} \right) + \frac{b+n}{w+b+n} \left(\frac{b}{w+b} \right)} = \frac{w}{w+b+n}$$

2.156 Note that $S = S \cup \emptyset$, and S and \emptyset are disjoint. So, $1 = P(S) = P(S) + P(\emptyset)$ and therefore $P(\emptyset) = 0.$

2.157 There are 10 nondefective and 2 defective tubes that have been drawn from the machine, and number of distinct arrangements is $\binom{12}{2} = 66$.

- a. The probability of observing the specific arrangement is $1/66$.
- b. There are two such arrangements that consist of "runs." In addition to what was given in part (a), the other is *DDNNNNNNNNNN*. Thus, the probability of two runs is $2/66 = 1/33$.

2.158 We must find $P(R \leq 3) = P(R = 3) + P(R = 2)$, since the minimum value for R is 2. If the two D 's occurs on consecutive trials (but not in positions 1 and 2 or 11 and 12), there are 9 such arrangements. The only other case is a defective in position 1 and 12, so that (combining with Ex. 2.160 with $R = 2$), there are 12 possibilities. So, $P(R \leq 3) = 12/66$.

2.159 There are $9!$ ways for the attendant to park the cars. There are $3!$ ways to park the expensive cars together and there are 7 ways the expensive cars can be next to each other in the 9 spaces. So, the probability is $7(3!)/9! = 1/12$.

2.160 Let A be the event that current flows in design A and let B be defined similarly. Design A will function if (1 or 2) & (3 or 4) operate. Design B will function if (1 & 3) or (2 & 4) operate. Denote the event $R_i = \{\text{relay } i \text{ operates properly}\}$, $i = 1, 2, 3, 4$. So, using independence and the addition rule,

$$P(A) = (R_1 \cup R_2) \cap (R_3 \cup R_4) = (.9 + .9 - .9^2)(.9 + .9 - .9^2) = 0.9801.$$

$$P(B) = (R_1 \cap R_3) \cup (R_2 \cap R_4) = .9^2 + .9^2 - (.9^2)^2 = .9639.$$

So, design A has the higher probability.

2.161 Using the notation from Ex. 2.163, $P(R_1 \cap R_4 | A) = P(R_1 \cap R_4 \cap A) / P(A)$.

Note that $R_1 \cap R_4 \cap A = R_1 \cap R_4$, since the event $R_1 \cap R_4$ represents a path for the current to flow. The probability of this above event is $.9^2 = .81$, and the conditional probability is in question is $.81/.9801 = 0.8264$.

2.162 Using the notation from Ex. 2.163, $P(R_1 \cap R_4 | B) = P(R_1 \cap R_4 \cap B) / P(B)$.

$R_1 \cap R_4 \cap B = (R_1 \cap R_4) \cap (R_1 \cap R_3) \cup (R_2 \cap R_4) = (R_1 \cap R_4 \cap R_3) \cup (R_2 \cap R_4)$. The probability of the above event is $.9^3 + .9^2 - .9^4 = 0.8829$. So, the conditional probability in question is $.8829/.9639 = 0.916$.

2.163 There are $\binom{8}{4} = 70$ ways to choose the tires. If the best tire the customer has is ranked #3,

the other three tires are from ranks 4, 5, 6, 7, 8. There are $\binom{5}{3} = 10$ ways to select three tires from these five, so that the probability is $10/70 = 1/7$.

2.164 If $Y = 1$, the customer chose the best tire. There are $\binom{7}{3} = 35$ ways to choose the remaining tires, so $P(Y = 1) = 35/70 = .5$.

If $Y = 2$, the customer chose the second best tire. There are $\binom{6}{3} = 20$ ways to choose the remaining tires, so $P(Y = 2) = 20/70 = 2/7$. Using the same logic, $P(Y = 4) = 4/70$ and so $P(Y = 5) = 1/70$.

2.165 a. The two other tires picked by the customer must have ranks 4, 5, or 6. So, there are $\binom{3}{2} = 3$ ways to do this. So, the probability is $3/70$.

b. There are four ways the range can be 4: #1 to #5, #2 to #6, #3 to #7, and #4 to #8. Each has probability $3/70$ (as found in part **a**). So, $P(R = 4) = 12/70$.

c. Similar to parts **a** and **b**, $P(R = 3) = 5/70$, $P(R = 5) = 18/70$, $P(R = 6) = 20/70$, and $P(R = 7) = 15/70$.

2.166a. For each beer drinker, there are $4! = 24$ ways to rank the beers. So there are $24^3 = 13,824$ total sample points.

b. In order to achieve a combined score of 4 or less, the given beer may receive at most one score of two and the rest being one. Consider brand A. If a beer drinker assigns a one to A there are still $3! = 6$ ways to rank the other brands. So, there are 6^3 ways for brand A to be assigned all ones. Similarly, brand A can be assigned two ones and one two in $3(3!)^3$ ways. Thus, some beer may earn a total rank less than or equal to four in $4[6^3 + 3(3!)^3] = 3456$ ways. So, the probability is $3456/13824 = 0.25$.

2.167 There are $\binom{7}{3} = 35$ ways to select three names from seven. If the first name on the list is included, the other two names can be picked $\binom{6}{2} = 15$ ways. So, the probability is $15/35 = 3/7$.

2.168 It is stated that the probability that Skylab will hit someone is (unconditionally) $1/150$, without regard to where that person lives. If one wants to know the probability condition on living in a certain area, it is not possible to determine.

2.169 Only $P(A|B) + P(\bar{A}|B) = 1$ is true for any events A and B .

2.170 Define the events: D : item is defective C : item goes through inspection

Thus $P(D) = .1$, $P(C|D) = .6$, and $P(C|\bar{D}) = .2$. Thus,

$$P(D|C) = \frac{P(C|D)P(D)}{P(C|D)P(D) + P(C|\bar{D})P(\bar{D})} = .25.$$

2.171 Let A = athlete disqualified previously B = athlete disqualified next term

Then, we know $P(B|\bar{A}) = .15$, $P(B|A) = .5$, $P(A) = .3$. To find $P(B)$, use the law of total probability: $P(B) = .3(.5) + .7(.15) = 0.255$.

2.172 Note that $P(A) = P(B) = P(C) = .5$. But, $P(A \cap B \cap C) = P(HH) = .25 \neq (.5)^3$. So, they are not mutually independent.

2.173a.
$$\begin{aligned} P[(A \cup B) \cap C] &= P[(A \cap C) \cup (B \cap C)] = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \\ &= P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) = [P(A) + P(B) - P(A)P(B)]P(C) \\ &= P(A \cap B)P(C). \end{aligned}$$

b. Similar to part a above.

2.174a. $P(\text{no injury in 50 jumps}) = (49/50)^{50} = 0.364$.

b. $P(\text{at least one injury in 50 jumps}) = 1 - P(\text{no injury in 50 jumps}) = 0.636$.

c. $P(\text{no injury in } n \text{ jumps}) = (49/50)^n \geq 0.60$, so n is at most 25.

2.175 Define the events: E : person is exposed to the flu F : person gets the flu

Consider two employees, one of who is inoculated and one not. The probability of interest is the probability that at least one contracts the flu. Consider the complement:

$$P(\text{at least one gets the flu}) = 1 - P(\text{neither employee gets the flu}).$$

For the inoculated employee: $P(\bar{F}) = P(\bar{F} \cap E) + P(\bar{F} \cap \bar{E}) = .8(.6) + 1(.4) = 0.88$.

For the non-inoculated employee: $P(\bar{F}) = P(\bar{F} \cap E) + P(\bar{F} \cap \bar{E}) = .1(.6) + 1(.4) = 0.46$.

So, $P(\text{at least one gets the flu}) = 1 - .88(.46) = 0.5952$

2.176a. The gamblers break even if each win three times and lose three times. Considering the

possible sequences of “wins” and “losses”, there are $\binom{6}{3} = 20$ possible orderings. Since

each has probability $(\frac{1}{2})^6$, the probability of breaking even is $20(\frac{1}{2})^6 = 0.3125$.

b. In order for this event to occur, the gambler Jones must have \$11 at trial 9 and must win on trial 10. So, in the nine remaining trials, seven “wins” and two “losses” must be

placed. So, there are $\binom{9}{2} = 36$ ways to do this. However, this includes cases where

Jones would win before the 10th trial. Now, Jones can only win the game on an even trial (since he must gain \$6). Included in the 36 possibilities, there are three ways Jones could

win on trial 6: *WWWWWWLL*, *WWWWWWLLW*, *WWWWWWLWL*, and there are six ways Jones could win on trial 8: *LWWWWWWL*, *WLWWWWWWL*, *WWLWWWWL*, *WWWLWWL*, *WWWLWWL*, *WWWLWWL*. So, these nine cases must be removed from the 36. So, the probability is $27\left(\frac{1}{2}\right)^{10}$.

2.177a. If the patrolman starts in the center of the 16x16 square grid, there are 4^8 possible paths to take. Only four of these will result in reaching the boundary. Since all possible paths are equally likely, the probability is $4/4^8 = 1/4^7$.

b. Assume the patrolman begins by walking north. There are nine possible paths that will bring him back to the starting point: *NNSS*, *NSNS*, *NSSN*, *NESW*, *NWSE*, *NWES*, *NEWS*, *NSEW*, *NSWE*. By symmetry, there are nine possible paths for each of north, south, east, and west as the starting direction. Thus, there are 36 paths in total that result in returning to the starting point. So, the probability is $36/4^8 = 9/4^7$.

2.178 We will represent the n balls as 0's and create the N boxes by placing bars (|) between the 0's. For example if there are 6 balls and 4 boxes, the arrangement

0|00||000

represents one ball in box 1, two balls in box 2, no balls in box 3, and three balls in box 4. Note that six 0's were need but only 3 bars. In general, n 0's and $N - 1$ bars are needed to

represent each possible placement of n balls in N boxes. Thus, there are $\binom{N+n-1}{N-1}$

ways to arrange the 0's and bars. Now, if no two bars are placed next to each other, no box will be empty. So, the $N - 1$ bars must be placed in the $n - 1$ spaces between the 0's.

The total number of ways to do this is $\binom{n-1}{N-1}$, so that the probability is as given in the

problem.