Math 321
Lab #4

Note: answers may vary slightly due to rounding.

1. Big Grack's used car dealership reports that the probabilities of selling 1,2,3,4, and 5 cars in one week are 0.256, 0.239, 0.259, 0.185, and 0.061 respectively. Find the expected number of cars sold and the variance and standard deviation. (2.556,1.4949,1.2226)

2. Suppose that Y is a random variable that takes on one of 4 values, 1, 2, 3, 6. If p(y=1) = 0.2, and p(y=2) = 0.1, p(y=3) = .4,
   (a) what is p(y=6)? (.3)
   (b) Find the expected value, variance and standard deviation for Y. (3.4, 3.44, 1.8547)

3. When you give a casino \$5 for a bet on the number 7 in roulette, you have a 1/38 probability of winning \$175 plus the \$5 paid to play and a 37/38 probability of losing \$5.
   (a) Should you play this game? Why?(no, μ= -.26)
   (b) If you bet \$10 and the return is 35 times the original bet plus the original bet, what is your expected winnings? (-.53)
   (c) If you bet \$20 and the return is 35 times the original bet plus the original bet, what is your expected winnings? (-\$1.05)

4. Based on passed results, there is a 0.12 probability that the Stanley Cup final will last four games, a 0.253 probability that it will last five games, a 0.217 probability that it will last six games, and a 0.410 probability that it will last seven games.
   (a) Find the mean and standard deviation for the numbers of games that the Stanley Cup finals last. (5.917, 1.1361, 1.0659)
   (b) Is it unusual for a team to "sweep" by winning in four games? Why?

5. Each computer chip produced by machine A is defective with a probability of 0.1, whereas each computer chip produced by machine B is good with a probability of 0.95. 42% of computer chips are produced by machine A, the remainder by machine B. One chip is chosen at random from the batch that is made by machine A and 1 chip is chosen at random from the batch made by machine B. Let the random variable Y represent the number of defective chips. Determine the probability distribution of Y. Find the expected number of defective chips, the variance and standard deviation. (.15, .1375, .3708)

6. There are 3 boxes with balls inside each. You select a box at random, and then randomly select 3 balls from it. The first box contains 4 red balls, 4 blue balls, and 4 green balls; the second box contains 4 red balls, 5 blue balls, and 6 white; the third contains 1 yellow ball and six blue balls. Let the random variable Y represent the number of blue balls selected. Determine the probability distribution of Y. Find the expected number of blue balls, $E[Y^2]$,the variance and standard deviation. (1.5238, 3.3248, 1.0028, 1.0014)

7. Two fair dice are rolled. Let Y denote the larger of the two numbers appearing on the two dice (e.g. if a 2 and 3 are rolled, y = 3, if a 3 and 3 are rolled y = 3). Determine the probability distribution of Y, as well as the mean of Y and the standard deviation of Y. (4.4722, 1.9716,1.4042)

8. A hunter finds a crow sitting in a tree. He has only 3 shells in his shotgun. Assuming that the crow does not fly away after it is shot at, and the probability that the hunter hits the crow on each shot is 0.4, find the probability distribution of the number of shots fired at the crow. In addition, find the expected number of shots fired at the crow as well as the variance of the number of shots fired at the crow. (1.96), (0.7584)

9. A hunter finds a crow sitting in a tree. The probability that the hunter hits the crow is .4. If the crow is missed, there is an 80% chance that it is scared away by the gunshot. The hunter has 3 shells and he shoots until the crow is hit or it flies away. Find the expected number of shots fired and the standard deviation of the number of shots fired. (1.134), (0.3810)

10. A contractor will bid for 2 jobs in sequence. She has a probability of 0.5 of winning the first job. If she wins the first job, then she has a 0.2 chance of winning the second job: if she loses the first job, then she has a 0.4 chance of winning the second job. Let Y denote the number of jobs she wins. Find the probability distribution of Y, as well as E[Y], E[Y²], V[Y]. (0.8, 1.0, 0.36)

11. A casino owner decided to charge $2 to play the following game. 3 cards are drawn from a deck of 52. If no hearts are drawn the player gets nothing: if 1 heart is drawn the player wins $1; if 2 hearts are drawn the player wins $5. How high should the owner make the jackpot on a pull of 3 hearts if he wants the "**house**" to average 50 cents a play? ($28.93)

12. A game of chance is considered **fair** if a player's expectation is equal to zero. If someone pays me $10 each time I roll a 1,2,3 or 4 with a balanced die, how much should I pay them when I roll a 5, or 6 to make the game fair? ($20)

Minitab instructions for a Binomial probability distribution.
Our main interest will be in the DATA window, which serves as the worksheet for entering data, and the MENU bar that allows the selection of commands to perform various tasks. The results will be presented in the SESSION window.

Eg. The probability that an elderly person who has high blood pressure will get a heart attack is 0.3. Find the probability that of 5 elderly people,
(a) none will get a heart attack (b) not more than one will get a heart attack (c) at least 3 will get a heart attack.

For question 1 n = 5, we let the random variable (Y) represent the number out of 5 who get a heart attack, p = .3, q = .7

(a) p(0) = $\binom{5}{0}(.3)^0(.7)^5 = .16807$ by hand

using the computer: From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL...... a dialog box appears.
If a single probability is needed as in part a) of the example:
Select **PROBABILITY** from the options listed (clicking on the circle, enters a dot.)
In the box by NUMBER OF TRIALS, type the number **5**.
In PROBABILITY OF SUCCESS, insert **0.3**.
As INPUT CONSTANT, specify the "x" value from the question (**0** for part (a)). Click on OK
The probability then comes up on the SEESION screen. [0.16807]

(b) p(Y≤1) = p(y=0) +p(y=1) = $\binom{5}{0}(.3)^0(.7)^5 + \binom{5}{1}(.3)^1(.7)^4 = .52822$

Using the computer: When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>BINOM}
Select **CUMULATIVE PROBABILITY** from the options.
Enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS **(5 and 0.3)** as before.
As INPUT CONSTANT, specify the "x" from the question (**1** for part (b)). Click on OK
The number appearing on the SESSION screen will be the p(Y ≤ y). {p(Y ≤ 1) for b)}
[0.52822]

(c) p(Y≥3) = p(y=3)+p(y=4) +p(y=5) = .16308 or
p(Y≥3)= 1-p(Y≤2) = 1-[p(y=0) +p(y=1) +p(y=2)] = 1-.83692 = .16308

Using the computer: the required probabilities would have to be expressed in terms of the cumulative probability and the complement........p(Y ≥ 3)= 1-P(Y ≤ 2) The instructions are exactly the same as the previous except in **input constant** enter 2 because we want the cumulative probability up to and including 2. The computer will give you .83692. You then have to subtract this from 1 to get the final answer. [0.16308]

When several different questions are asked about the same distribution, it might be helpful to have all of the probabilities for each individual "x" calculated at once if n is not very large.

Enter the values 0,1,2,3,4,5 into c1, the first column of the worksheet.
From the menu bar, select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL; select PROBABILITY from options; enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS (5 and 0.3).In the box associated with INPUT COMUMN indicate C1 (where you have listed the possible values of X) and request OPTIONAL STORAGE in C2. CLICK on OK>On the worksheet, the probabilities for each value of X appear in column C2.

Repeating the procedure with CUMULATIVE PROBABILITY and choosing OPTIONAL STORAGE in C3 will complete a table of x values, individual probabilities, and cumulative probabilities in the WORKSHEET.

1. Assume that male and female births are equally likely and that the birth of any child does not affect the probability of the gender of any other children.
   (a) Find the probability of
      (i) Exactly 4 girls in 10 births. (.2051)
      (ii) At least 4 girls in 10 births. (.8281)
      (iii) Exactly 8 girls in 20 births. (.1201)
      (iv) At most 7 girls in 20 births. (.1316)
      (v) More than 10 girls. (.4119)
   (b) Find the expected number of girls and its variance for
      (i) 10 births. (5, 2.5)
      (ii) 20 births. (10, 5)

2. According to a market-share study, 13% of televisions is use are tuned to *Hockey Night in Canada* on Saturday night. Assume that an advertiser wanted to verify that 13% market share value by conducting its own survey, and a pilot survey begins with 50 households having TV sets in use at the time of *Hockey Night in Canada* broadcast.
   (a) Find the probability that none of the households are tuned to *Hockey Night in Canada.* (.000946)
   (b) Find the probability that at least 6 of the households are tuned to *Hockey Night in Canada.* (.6463)
   (c) If 7 households are observed to have tuned to *Hockey Night in Canada,* does this support the earlier findings? Why or why not?

3. In a survey, the Canadian Automobile Association (CAA) found that 6.1% of its members bought their cars at a used-car lot.
(a) If 15 CAA members are selected at random, what is the probability that
      (i) 4 of them bought their cars at a used-car lot. (.0095)
      (ii) at least 2 bought their cars at a used-car lot. (.2319)
(b) If 300 members are selected at random, how many are expected to have bought their cars at a used-car lot? (18.3)

4. The captain of a Navy gunboat orders a volley of 26 missiles to be fired at random along a 500ft stretch of shoreline that he hopes to establish as a beachhead. Dug into the beach is a 30 foot-long bunker serving as the enemy's first line of defence. What is the probability that exactly
(a) 3 missiles will hit the bunker? (0.1353)
(b) Between 6 and 9 missiles, inclusive, will hit the bunker? (0.00376)
(c) If at least 10 successful missiles are needed to destroy the bunker, what is the probability that the captain is unsuccessful in destroying the bunker? (~1.00)
(d) What is the expected number of missiles to hit the bunkerk? (1.56)

5. Prove that $\mu = np$ and $\sigma^2 = npq$ for a binomial distribution.

6. Suppose that 10% of the CDs produced are defective. A manager of a CD manufacturing plants what to test a few CD's from the production line.

(a) Find the probability that
   (i) the first defective will be the 3rd CD he checks. (.081)
   (ii) the first defective CD will be the 4th CD he checks. (.0729)
   (iii) the first defective CD will be on or after the 4th CD. (.729)
   (iv) the third defective CD will be the 10th CD that he checks. (.0172)
   (v) the fourth defective CD will be the 10th CD that he checks. (.0045)

(c) How many CDs is he expected to inspect on average until he finds a defective one? (10)
(d) How many CDs is he expected to inspect on average until he finds the third defective one? (30)
(e) How many CDs is he expected to inspect on average until he finds the fourth defective one? (40)

7. The World Series is a best of 7 game series (whoever wins 4 games first will win the series) between Team A and Team B. The probability that Team A wins any game is 0.6. There are no ties in the World Series. Assuming the outcome of each game is independent of what has happened in prior games.

(a) What is the probability that Team A wins the World Series? (0.7102)
(b) Given that Team A has won the first two games, what is the probability that the World Series will end in 6 games? (0.1984)

Minitab instructions for the Hypergeometric Probability distribution

The instructions are very similar to the binomial instructions.
From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>Hypergeometric...... a dialog box appears. There are various ways in which you can find the required answers

i. If a single probability is needed:
   Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
   In the box by N type the total number of available items
   In the box by M (which is r in our formula) type the total number of available successes
   In the box by n, type the total number of items you are selecting.
   As INPUT CONSTANT, specify the "y" number of successes out of n items. Click on OK

ii. If a When a sum of probabilities is involved, begin in the same way {CALC>PROB DISTR>Hypergeometric}
   Select CUMULATIVE PROBABILITY from the options.
   Enter a value in the N box, M box and n box.
   As INPUT CONSTANT, specify the "y" (number of successes in n trial). Click on OK
   The number appearing on the SESSION screen will be the $P(Y \le y)$.

8. Suppose 4 fuses are selected from a batch of 20 where 6 are known to be defective. Find the

(a) probability that only 1 defective fuse is selected. (.4508)
(b) probability that at least 1 defective fuse is selected. (.7934)
(c) expected number of defective fuses out of 4. (1.2)
(d) variance of defective fuses out of 4. (.7074)
(e) Expected number of defective fuses if 5 are selected. (1.5)

9. An urn contains 8 white balls and 4 red balls. Three balls are selected at random from the urn. Find the mean and standard deviation of the probability distribution for the number of red balls. (1.000) , (0.7385)

Minitab instructions for the Poisson Probability distribution

The Poisson instructions are similar to the binomial.

10. The number of mistakes in one page of a solutions manual to a statistics textbook follows a Poisson distribution with a rate of 2.2 mistakes per page.

(a) Find the probability that a randomly chosen page contains exactly 3 mistakes.

(b) Find the probability that a randomly chosen page contains at most 4 mistakes.
(c) Find the probability that a randomly chosen page contains at least 7 mistakes.

From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>POISSON...... a dialog box appears. There are various ways in which you can find the required answers

i. If a single probability is needed as in part a) of the example:
   Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
   In the box by MEAN, type 2.2
   As INPUT CONSTANT, specify the "y" value (# of successes) from the question (**3** for part a)). Click on OK
   The probability then comes up on the SEESION screen. [0.1966]

ii. When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>POISSON}
   Select CUMULATIVE PROBABILITY from the options.
   Enter MEAN 2.2.
   As INPUT CONSTANT, specify the "y" from the question (**4** for part b)). Click on OK
   The number appearing on the SESSION screen will be the $P(Y \le y)$. {$P(Y \le 4)$ for b)}
   [0.9275]

This method could also be used for part c), but the required probabilities would have to be expressed in terms of the cumulative probability and the complement........$P(Y \ge 7)= 1-P(Y \le 6)$ [0.0075]

11. The number of cars arriving at the service station just down the street from my place follows a Poisson distribution with a rate of 12 per hour. Find the probability that within the next hour

(a) at most 10 cars will arive. (0.3472)
(b) exactly 15 will arrive. (0.0724)
(c) At least 2 will arive.(0.9999)

12. A data processing company uses 200 personal computers. The probability that any one of them will break on a given day is 0.01. There are 3 spare computers availbable and broken machines are always fixed overnight. Using the Poisson distribution, find the probability that:

(a) on a given day, all computers will be used.(0.1804)
(b) On given day the number of spare computers is sufficient to replace all the broken ones. (0.8571)

13. A medical drop-in clinic receives, on average, 5 patients per hour. Find the probabilitiies of the following events:

(a) at least 2 patients will arive in a one hour period? (.9596)
(b) 9 patients will arrive in 90 minute period? (0.1144)

14. Telephone calls come through an exchange at a rate of 2.6 call per 10 minute interval. What is the probability that

(a) exactly 2 calls in a **5 minute** interval? (0.2303)
(b) at least 2 calls in a 10 minute interval? (0.7326)
(c) no more than 1 call in a 1 minute interval? (.9715)

15. The number of mistakes in one page of a solutions manual to a statistics textbook follows a Poisson distribution with a rate of 2.2 mistakes per page. Find the probability that a randomly chosen page contains at most 4 mistakes.(.9275)

Do all the questions in the text for section 3.1-3.8 plus the supplementary questions dealing with these topics.