Math 321
Lab #5

Note: answers may vary slightly due to rounding.

Normal probability distribution

Go to the MINITAB program
MINITAB INSTRUCTIONS
CALC ⇒Probability Distributions⇒normal

Finding the area above and below a Z-value under the Standard Normal Curve on the computer.
For a given Z-value, we want to find a probability. In the dialog box, which corresponds to the Normal distribution, you have three choices:

Probability
Cumulative probability
Inverse cumulative probability

Click **Cumulative probability**. This will calculate the cumulative probability associated with a specific Z-value (or the area under the Standard Normal Curve to the **left** of a specific Z-value.)
The middle of the dialog box has 2 options:

Mean
Standard Deviation

The default values for the **Mean** and **Standard Deviation** are **0** and **1**, respectively. There is no need to change these values, so just leave these as is.

In the bottom portion of the dialog box, select **Input constant.** It is in this box that you enter a specific Z-value. Once you have completed this, either press return or "click" on **OK**. In the upper portion of your screen, or the command module, MINITAB will return the area to the **left** of the Z-value you have entered. Note that when this routine is employed, the probability returned is **ALWAYS THE AREA TO THE LEFT OF** the Z-value entered above or $P(Z<z)$

For practice, try question 1 using this routine.

1. Given that Z is a standard normal random variable, compute the following probabilities:

(a) $P(Z \leq -2.44)$ (.0073)
(b) $P(Z \geq 1.62)$ (.0526)
(c) $P(Z \geq -1.38)$ (.9162)
(d) $P(-0.72 \leq Z \leq 0)$ (.2642)
(e) $P(-0.62 \leq Z \leq 0.62)$ (.4648)
(f) $P(-0.45 \leq Z \leq 2.11)$ (.6562)
(g) $P(0.34 \leq Z \leq 2.33)$ (.3570)

Finding a Z-value for a given area (or probability under the Standard Normal Curve)
For a given probability, you are required to find a Z-value that corresponds to this probability. This requires the use of the **Inverse cumulative probability** routine in the dialog box employed above.

"Click" on the circle which corresponds to **Inverse cumulative probability** and just as was done previously, do not touch the box labeled **Mean** and **Standard Deviation**.

This routine needs an area, and will subsequently find the Z-value which matches up with the area entered. Just as was done above, move your mouse down to the bottom portion of the dialog box and "click" on the circle which corresponds to **Input constant**. Previously you entered a Z-value here. But now you want to find a Z-value for a given area, or probability. So the number you will enter in the **Input constant** box is a probability, or an area to the left of the Z-value in question. Once you have entered the correct probability, either press return or "click" on **OK.** MINITAB will return a Z-value in the command module on the upper portion of your screen.

A good rule-of-thumb in these types of problems is to draw your standard normal curve and piece together the areas given. The Z-value will be given when you specify the area to the left of that value. Practice this routine on question 2.

2. Given that Z is a standard normal random variable, determine ***Zo*** if it is known that:

(a) $P(-Zo \le Z \le Zo) = 0.90$ (1.645)
(b) $P(-Zo \le Z \le Zo) = 0.10$ (.1257)
(c) $P(Z \ge Zo) = 0.20$ (.842)
(d) $P(-1.66 \le Z \le Zo) = 0.25$ (-.529)
(e) $P(Z \le Zo) = 0.40$ (-.253)
(f) $P(Zo \le Z \le 1.80) = 0.20$ (.720)

3. Assume that the height of male college students are normally distributed with a mean of 178.05 cm and a standard deviation of 6.86 cm.
(a) Find the percentage of male students who fall between the Toronto Maple Leaf's average height of 183.9 cm and the Philadelphia Flyers' average height of 189.48 (15.02%)
(b) If the question in (a) was changed to inches (conversion is to divide cm by 2.54), would the answer change?
(c) Among 500 randomly selected male college students, how many would fall between the interval in (b)? (75.1)

4. Find the percentage of years in which Canadian imports from the Middle East and Africa are between \$339 million and \$8004 million. Assume that the annual import values exhibit no trend, and are normally distributed with a mean of \$2502 million and a standard deviation of \$905 million (99.15%)

5. Assume that human body temperatures are normally distributed with a mean of 36.4°C and a standard deviation of 0.62°C.
(a) If we define a fever to be a body temperature above 37.8°C, what percentage of normal and healthy persons would be considered to have a fever? Does this percentage suggest that a cut-off of 37.8°C is appropriate? (1.19%)
(b) If we defined a fever so that only 0.5% of the population would have a body temperature above a certain temperature, what is this temperature? (37.997°C)

6. The time it takes Bob to get from home to his office follows a normal distribution. The probability that it takes him less than 3 minutes is 0.345. The probability that it takes him more than 10 minutes is 0.01. Find the average time and variance (μ and σ^2) of this normal distribution. (~4.0256, ~6.5746)

7. It takes on average 12.3 minutes to run a race with a standard deviation of 0.4 minutes.
(a) What is the probability that the runner will take between 12.1 and 12.6 minutes to finish the race? (.4649)
(b) What is the maximum time (in minutes) the runner must have for the time to be classified "among the fastest 5% of his times"? (11.642 min)

8. Find the mean and variance, using $E[Y]=\sum yp(y)$, for discrete and $E[Y]=\int yf(y)dy$ for continuous and $V[Y]= E[Y^2]-E[Y]^2$ for both discrete and continuous if they exist for each of the following distributions:

A $f(y) = \frac{3!}{y!(3-y)!}\left(\frac{1}{2}\right)^3$ y = 0,1,2,3, zero elsewhere [1.5, .75]

B $f(y) = 6y(1-y)$ 0<y<1, zero elsewhere [.5,.05]

C $f(y) = \frac{2}{y^3}$ 1<y<∞, zero elsewhere [2, does not exist]

9. Show that $\mu = E[Y] = \frac{\theta_1 + \theta_2}{2}$ $\sigma^2 = V[Y] = \frac{(\theta_2 - \theta_1)^2}{12}$ for a uniform distribution.

10. Evaluate $\int_0^\infty (1/54)y^2 e^{-y/3}dy$ (hint: integration by parts) [1]

11. Find the moment generating function for the following distribution function and find E[Y] and V[Y] by using the m.g.f.

    f(y) = e^{-y} $y > 0$ (m(t) = $\frac{1}{1-t}$ $|t| < 1$, E[Y] = 1, V[Y] = 1)

12. Find the m.g.f for an exponential distribution and find the E[Y] and V[Y] by using the mgf. (m(t) = $(1-\beta t)^{-\alpha}$ E[Y] = β V[Y] =β²)

13. show that the moment generating function for the random variable Y having the probability density function f(y) = 1/3, -1<y<2, zero elsewhere is

    $$m(t) = \frac{e^{2t} - e^{-t}}{3t}, \ t \neq 0$$

14. Find the moment generating function for $f(y) = \frac{3!}{y!(3-y)!}\left(\frac{1}{2}\right)^3$ y = 0,1,2,3, zero elsewhere and use it to find the mean and variance. [1.5,.75]

15. Do all the questions in section 3.9, omit section 3.10. Do all the question in chapter 4 excluding those that deal with section 4.7 and 4.11.