Math 321
Lab7
Hypothesis Tests, Type I, Type II error

Note: Answers may vary slightly due to rounding.

1. The manufacturer of an over-the-counter pain reliever claims that its product brings pain relief to headache suffers in less than 3.5 minutes, on average. In order to be able to make this claim in its television advertisements, the manufacturer was required by a particular television network to present statistical evidence in support of the claim. The manufacturer reported that for a random sample of 50 headache sufferers, the mean time to relief was 3.3 minutes and the standard deviation was 66 seconds.
   (a) Does this data support the manufacturer's claim? Test using a 5% significance level. [Ho: μ=3.5, Ha: μ<3.5, z-calc=-1.2856 , zcritical= -1.645, No]
   (b) Find the p-value. [.0993]
   (c) What type of error could have been made?
   (d) In general, do large p-values or small p-values support the manufacturer's claim? Explain.
   (e) At what levels of significance would you come to a different conclusion in (a)? [α >.0993]

2. The average total blood protein in a healthy adult is 7.25 grams per decilitre. A series of 8 blood tests were run on a particular patient over several days giving the following results:
   7.23 7.25 7.28 7.29 7.32 7.26 7.27 7.24
   (a) At a 5% significance level, do these test results indicate this patient has the total blood protein level of a healthy adult? [Ho: μ=7.25, Ha: $\mu \neq$ 7.25, tcalc = 1.6951, tcrit = ±2.365 ,yes]
   (b) Find the p-value. [0.134]
   (c) What type of error could have been made?
   (d) What assumptions if any were made?

3. In a survey, 1039 adults were asked " How much respect and confidence do you have in the public school system?" The results, reported in the Toronto Star (Sept. 26, 1988), are shown below:

Responses	A great deal	Quite a lot	Some	Very little	No opinion
Percentages	12%	30%	35%	13%	10%

   (a) It is believed that at least 50 % of people have a "great deal" or "quite a lot" of respect for the public school system. Test this belief at the 5% significance level. [Ho: p≥5, Ha: p<.5, zcalc = -5.157, zcrit= -1.645]
   (b) Find the p-value. [~0]
   (c) What type of error could have been committed?

4. The effects of drug and alcohol on the nervous system have been the subject of considerable research. Suppose a research neurologist is testing the effects of a drug on response time by injecting 100 rats with a unit dose of the drug, subjecting each to a neurological stimulus, and recording its response time. The neurologist knows the response time for rats not infected with the drug (the control mean) is 1.2 seconds. She wishes to test whether the response time for drug-injected rats differs from that control mean. Her sample of 100 rats give a mean of 1.05 seconds and a standard deviation for 0.5 seconds.
   (a) Perform a test of her hypothesis at a 1% significance level. [Ho: μ=1.2, Ha: $\mu \neq$ 1.2, zcalc-3.0, zcrit= ±2.575, RHo]
   (b) Find the p-value. [.0026]
   (c) What type of error could have been committed?
   (d) At what levels of significance would you come to a different conclusion in (a)? [α <.0026(using z)]

5. Of the 200 individuals interviewed, 80 said they were concerned about fluorocarbon emissions in the atmosphere. It's believed that the majority (50% or more) of individuals are concerned about fluorocarbon emissions in the atmosphere.
   (a) Test this claim at the 1% significance level. [Ho: p≥.5, Ha: p<.5, zcalc-2.83, zcrit =-2.33, RHo]
   (b) Find the p-value. [.0023]
   (c) What type of error could have been committed?

(d) At what levels of significance would you come to a different conclusion in (a)? [α<.0023]

6. The reputations (and hence the sales) of many businesses can be severely damaged by shipments of manufactured items that contain an unusually large percentage of defectives. A manufacturer of alkaline batteries wants to be reasonably certain that fewer than 5% of its batteries are defective. Suppose 300 batteries are randomly selected from a very large shipment. Each is tested and 10 defective batteries are found.
   (a) Will this sample provide sufficient evidence to the manufacturer that this shipment will be satisfactory at a 1% significance level?
   [Ho: p=.05, Ha: p<.05, zcalc=-1.3272, zcrit = -2.33, No]
   (b) If α =.05, what would the conclusion be?
   (c) Find the p-value. [.0922]
   (d) What type of error could have been made?
   (e) At what levels of significance would you come to a different conclusion in (a)? [α>.0922]

7. A sporting goods manufacturer who produces both white and yellow tennis balls claims that more than 75% of all tennis balls sold are yellow. A marketing study of the purchases of white and yellow tennis balls at a number of stores showed that of 470 cans sold, 410 were yellow and 60 were white.
   (a) Is there sufficient evidence to support the manufacturer's claim at a 1% significance level?
   [Ho: p=.75, Ha: p>.75zcalc =6.1213, zcrit = 2.33, yes]
   (b) Find the p-value. [~0]
   (c) What type of error could have been committed?
   (d) Calculate the probability of a Type II error if, in fact, 80% of the tennis balls sold are yellow.
   [$\hat{p}_c$ =.7965, P(z>-.1897) = .4247]

8. Consider the following hypothesis test:

   Ho:μ = 2400

   Ha:μ > 2400

   The rejection region has been defined as R: Sample mean > 2446.5

   Complete the table below indicating whether the given (population mean, sample mean) pair would result in acceptance or rejection of the null hypothesis and which type of error, if any, would result.

Population Mean	Sample Mean	Ho True or False	Fail to Reject Ho or Reject Ho	Type of Error
2401	2400			
2401	2450			
2400	2450			
2400	2401			

9. Management of a shopping centre believes that on weekends, people spend more than an hour and a half on average at the centre. To see if this is the case, a parking survey will be taken for a random sample of 100 cars on weekends. It is assumed that the standard deviation of parking times is 30 minutes. Assume that testing is carried out at the 2.5% significance level. Determine the power of the test if the true mean parking time (for the population) is 97 minutes. [Ho: μ=90min, Ha: μ>90min
   $\bar{x}_c$ =95.88, P(z>-.3733) =0.6443]

10. The manufacturer of Grin toothpaste claims that children under the age of 10 years that use their toothpaste regularly have, on average, less than 2 cavities. A random sample of 25 children had an average of 1.94 cavities with a sample standard deviation of 0.13. Assume cavity rate is normally distributed. Do the data support the manufacturer's claim?
    (a) Carry out the test with a 5% significance level
    [Ho: μ =2, Ha: μ <2, tcalc=-2.3077, tcrit = -1.711, Rho]
    (b) Determine the p-value for the test in (a) [0.0150]
    (c) Using a 5% significance level, find the probability of making a type II error if the true mean for cavities is 1.96. [$\bar{x}_c$ =1.9555, P(t>-.1731) =0.5678]

12. A random sample of 324 adults shows that 120 smoke. At the 1% significance level
    (a) test the claim that more than 1/3 of all adults smoke. [Ho: p =1/3, Ha: p>1/3, zcalc = 1.4167, zcrit = 2.33, Fail to RHo]
    (b) What's the probability of making a type I error? (0.01)
    (c) Using a 1% significance level, find the power of the test if the true population proportion was 40%. [$\hat{p}_c$ = .3943, P(z>-.2094) = 0.5932]

13. Assume that you are using a significance level of α= 0.05 to test the claim that $\mu < 2$ and that your sample consists of 50 randomly selected values.
    (a) Find β given that the population actually has a normal distribution with $\mu = 1.5$ and $\sigma = 1$. [Ho: μ =2, Ha: μ <2, $\overline{x}_c$ =1.7674, P(z>1.8695) =0.0294]
    (b) Find the power of the test and describe what it means. [0.9706]