



Mathematics 327

Number Theory

(see Course Descriptions under the year applicable: <http://www.ucalgary.ca/pubs/calendar/>)

**Reference Text:** "Fundamental Number Theory with Applications," R.A. Mollin, CRC Press, Boca Raton, New York, London, Tokyo, (1997). (not necessarily a required text)

## *Syllabus*

### Topics

- Ch. 1 Arithmetic of the Integers: The fundamental laws. Divisibility. Prime Numbers. Applications to Computer Science.
- Ch. 2 Congruences: Basics. Linear Congruences. Arithmetic functions. The Chinese remainder theorem. Polynomial congruences.
- Ch. 3 Primitive Roots: Order. Existence. Indices. Applications to cryptography.
- Ch. 4 Quadratic Residues: Quadratic reciprocity law. Jacobi and Kronecker symbols. Quadratic polynomials and primes. Applications to primality testing.
- Ch. 5 Continued Fractions: Finite continued fractions. Infinite continued fractions. Periodic continued fractions. Continued fractions and factoring.
- Ch. 6 Diophantine Equations: Sums of squares. The equation  $x^2 - Dy^2 = n$ . Diophantine equations of higher degree. Elliptic curves, factoring and primality testing.

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# PMAT 427 Course Outcomes

## Elementary Number Theory

1. Articulate the rich history and current applicability of number theory.
2. Apply number theoretic algorithms to solve problems involving the integers.
3. Explain properties and significance of prime numbers and unique factorization.
4. Determine the structure of modular unit groups and apply number theoretic algorithms to compute their invariants.
5. Define the notion of a quadratic residue and their associated symbols (Legendre, Jacobi). Be able to perform efficient computation of these symbols using number theoretic algorithms and identifying the applicability of this symbols to solving certain problems. Demonstrate how the law of quadratic residuosity is the key input in these calculations.
6. Describe how continued fractions give a systematic way of describing real numbers while emphasizing rationality, or the deviation from it. Use continued fractions to be able to solve problems in Diophantine Analysis.
7. Use mathematical reasoning to establish the validity of mathematical statements.

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2017/11/09  
RJS  
Course outcomes added