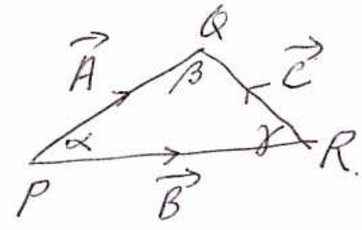
MATH 331 (L20)
Assignment 2 (Solution)

1. Pg 31.



6(a)

$$\vec{A} = (3,-4,-4)-(2,-1,+1) = (1,-3,-5)$$

$$\vec{B} = (1,-3,-5)-(2,-1,1) = (-1,-2,-6)$$

$$C = (3,-4,-4)-(1,-3,-5) = (2,-1,1)$$

$P=(2,-1,1)$
$Q=(3,-4,-4)$
$R=(1,-3,-5)$

$$\vec{A}\cdot\vec{B} = \|\vec{A}\|\|\vec{B}\|\cos\theta \quad \therefore \cos\theta = \frac{(1,-3,-5)\cdot(-1,-2,-6)}{\sqrt{1^2+3^2+5^2}\sqrt{1+4+36}}$$

$$\therefore \theta = \cos^{-1}\frac{35}{\sqrt{35}\sqrt{41}} = \cos^{-1}\sqrt{\frac{35}{41}}.$$

$$\vec{A}\cdot\vec{C} = \|\vec{A}\|\|\vec{C}\|\cos\beta$$

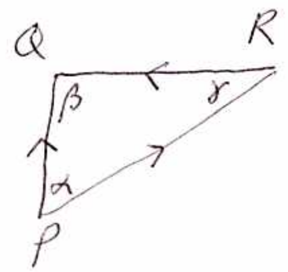
$$\therefore \cos\beta = \frac{(1,-3,-5)\cdot(2,-1,1)}{\sqrt{35}\sqrt{2^2+1+1}}$$

$$\therefore \beta = \cos^{-1}\frac{0}{\sqrt{35}\sqrt{6}} = \cos^{-1}0 = \frac{\pi}{2}$$

Note: $-\vec{B}\cdot\vec{C} = \|\vec{B}\|\|\vec{C}\|\cos\gamma$

$$\therefore \cos\gamma = \frac{-(-1,-2,-6)\cdot(2,-1,1)}{\sqrt{41}\sqrt{6}} = \frac{6}{\sqrt{41}\sqrt{6}} \quad \therefore \gamma = \cos^{-1}\sqrt{\frac{6}{41}}$$

So the Δ looks like:



8(a)-(c) $\|A+B\|^2 = (A+B)\cdot(A+B) = A\cdot A + 2A\cdot B + B\cdot B.$

$\|A-B\|^2 = (A-B)\cdot(A-B) = A\cdot A - 2A\cdot B + B\cdot B$

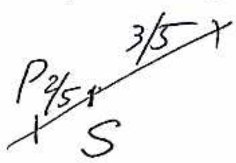
$\therefore \|A+B\|^2 + \|A-B\|^2 = 2\|A\|^2 + 2\|B\|^2.$

$\therefore \|A+B\|^2 - \|A-B\|^2 = 4A\cdot B.$

10. Let A be vector $(1,1,0)$

& $B = (1,1,1)$ & $C = (-1,3,2)$.

Then $A\cdot B = 2 = A\cdot C$. But $B \neq C$.



Pg 36, #4 (d)

4(a)-(c) are similar

$S\left(\frac{2}{5}\right) = P + \frac{2}{5}(Q-P)$

$= \frac{3}{5}P + \frac{2}{5}Q$

$= \left(-1, \frac{19}{5}, \frac{1}{5}\right).$

$P = (1,3,-1)$

$Q = (-4,5,2)$

Pg 42. #1.

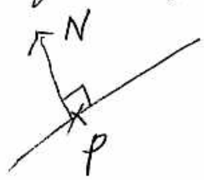
Line 1: $2x+3y=1 \rightarrow A = (2,3)$

Line 2: $5x-5y=7 \rightarrow B = (5,-5)$

Since $A\cdot B = 10-15 = -5 \neq 0$,

$\therefore$ The lines are not $\perp^r$.

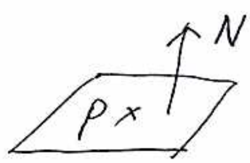
#4. Eq of line $\perp^r$ to $N=(-5,4)$ and passing through Pt $P(3,2)$ is



$X\cdot N = P\cdot N$

$-5x+4y = - \quad +8 = -7.$

Pg 43 #7(b) Eq of plane $\perp^r$ to $N=(-3,-2,4)$ & passing through $P(2,\pi,-5)$ is



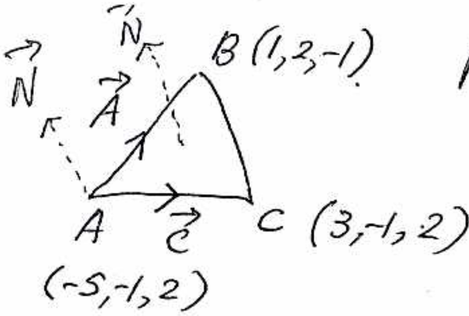
$X\cdot N = P\cdot N$

ie $-3x-2y+4z = -6-2\pi-20 = -26-2\pi.$

or $3x+2y-4z = 26+2\pi$

#8(c). $\vec{A} = (1,2,-1) - (-5,-1,2)$

$= (6,3,-3)$



$\vec{C} = (3,-1,2) - (-5,-1,2)$

$= (8,0,0)$

$$\therefore \vec{N} = \vec{A} \times \vec{C} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 6 & 3 & -3 \\ 8 & 0 & 0 \end{vmatrix} = (0, -24, -24)$$

or $\underline{\vec{N} = (0,1,1)}$.

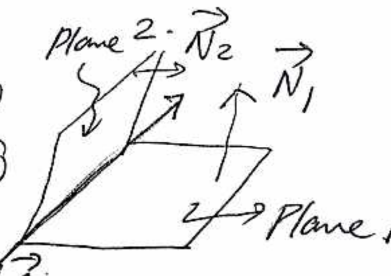
$\therefore$ Eq of plane passing through the 3 pts is

$X \cdot \vec{N} = A \cdot \vec{N}$ (or $B \cdot \vec{N}$ or $C \cdot \vec{N}$)

i.e. $y + z = (-5,-1,2)\cdot(0,1,1) = 1$.

#10. Plane 1 $\quad 2x - y + z = 1$ ——— ①

" 2 $\quad 3x + y + z = 2$ ——— ②



Method 1

The line of intersection is $\perp^r$ to both $\vec{N}_1 + \vec{N}_2$ (the normal vectors).

$$\vec{A} = \vec{N}_1 \times \vec{N}_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{vmatrix} = (-2, 1, 5)$$

is the vector // to the line of intersection.

Method 2.

① + ② : $\quad 5x + 2z = 3$.

Let $\quad z = t$

$\therefore 5x = 3 - 2z = 3 - 2t$.

$x = \frac{3}{5} - \frac{2}{5}t$.

$\therefore \vec{A} = \left(-\frac{2}{5}, \frac{1}{5}, 1\right)$. $\qquad \therefore y = 2x + z - 1 = 2\left(\frac{3}{5} - \frac{2}{5}t\right) + t - 1$

$= \underline{\frac{1}{5} + \frac{t}{5}}$.

#13(c) The angle between 2 planes is also the angle between their normal vectors:

$$N_1 = (1,2,-1) \text{ for plane } x+2y-z=1$$
$$N_2 = (-1,3,1) \quad ---- \quad -x+3y+z=2$$

$$\therefore \cos\theta = \frac{N_1 \cdot N_2}{\|N_1\|\|N_2\|} = \frac{4}{\sqrt{6}\sqrt{11}} = \sqrt{\frac{2}{33}}.$$

#14(a). $P=(1,3,5)$, $\vec{A}=(-2,1,1)$

The line through P in the $\vec{A}$ direction is

$$X(t) = P + t\vec{A} = (1,3,5) + t(-2,1,1)$$

This line intersects the plane

$$2x+3y-z=1.$$

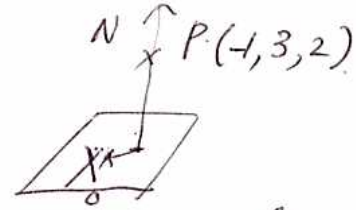
i.e. $$2(1-2t)+3(3+t)-(5+t)=1$$
$$6-2t=1$$
$$t=5/2.$$

$\therefore$ The pt of intersection is

$$(1,3,5)+\frac{5}{2}(-2,1,1)$$
$$=\left(-4, \frac{11}{2}, \frac{15}{2}\right).$$

(b)

Similar.



pg 43 # 16(b).

Take any pt on plane $2x-4y+z=1$, $(N=2,-4,1)$

$X_0 = (0,0,1)$.

Dist. from P to plane is $\dfrac{|(P-X_0)\cdot N|}{\|N\|}$

$$= \frac{|(-1,3,1)\cdot(2,-4,1)|}{\sqrt{4+16+1}} = \frac{13}{\sqrt{21}}$$

19. Plane $2x+4y-z=5 \longrightarrow N_1=(2,4,-1)$

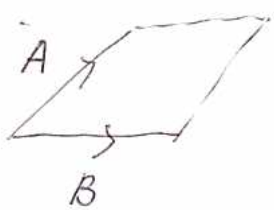
" $x-3y+2z=0 \longrightarrow N_2=(1,-3,2)$

The $\angle$ bet. the planes $=$ $\angle$ between N_1 + N_2.

$$\cos\theta = \frac{N_1\cdot N_2}{\|N_1\|\|N_2\|} = \frac{-12}{\sqrt{21}\sqrt{14}} = \frac{-12}{7\sqrt{6}} = \frac{-2\sqrt{6}}{7}$$

$$\theta = \cos^{-1} -\frac{2\sqrt{6}}{7}.$$

pg 48.
9(a)



Area of $\parallel$gram spanned by $A=(3,-2,4)$ + $B=(5,1,1)$ is $\|A\times B\|$.

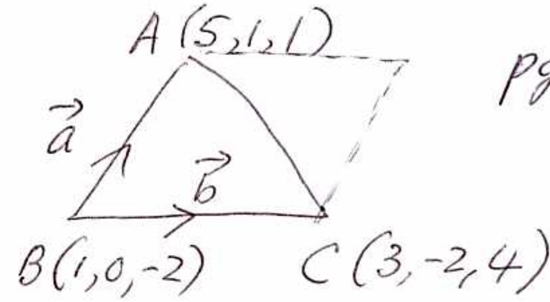
$$A\times B = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 3 & -2 & 4 \\ 5 & 1 & 1 \end{vmatrix} = (-6, 17, 13)$$

$\therefore$ Area $= \sqrt{36+17^2+169} = \sqrt{494}$ sq. units.

9(b) similar.

#7. $\vec{a} = (5,1,1) - (1,0,-2)$

$= (4,1,3)$



$\vec{b} = (3,-2,4) - (1,0,-2)$

$= (2,-2,6)$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 4 & 1 & 3 \\ 2 & -2 & 6 \end{vmatrix} = (12, -18, -10)$$

$\therefore$ Area of $\triangle ABC$ is $\frac{1}{2} \|\vec{a} \times \vec{b}\|$

$= \frac{1}{2} \sqrt{568} = \sqrt{142}$.