Assignment 2.

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NORM, $\text{Proj}_B A = \frac{A \cdot B}{\|B\|^2} B$

I, §4. EXERCISES

1. Find the norm of the vector A in the following cases.
   (a) $A = (2, -1)$, $B = (-1, 1)$
   (b) $A = (-1, 3)$, $B = (0, 4)$
   (c) $A = (2, -1, 5)$, $B = (-1, 1, 1)$
   (d) $A = (-1, -2, 3)$, $B = (-1, 3, -4)$
   (e) $A = (\pi, 3, -1)$, $B = (2\pi, -3, 7)$
   (f) $A = (15, -2, 4)$, $B = (\pi, 3, -1)$

2. Find the norm of vector B in the above cases.

3. Find the projection of A along B in the above cases.

4. Find the projection of B along A in the above cases.

5. Find the cosine between the following vectors A and B.
   (a) $A = (1, -2)$ and $B = (5, 3)$
   (b) $A = (-3, 4)$ and $B = (2, -1)$
   (c) $A = (1, -2, 3)$ and $B = (-3, 1, 5)$
   (d) $A = (-2, 1, 4)$ and $B = (-1, -1, 3)$
   (e) $A = (-1, 1, 0)$ and $B = (2, 1, -1)$

6. Determine the cosine of the angles of the triangle whose vertices are
   (a) $(2, -1, 1)$, $(1, -3, -5)$, $(3, -4, -4)$.
   (b) $(3, 1, 1)$, $(-1, 2, 1)$, $(2, -2, 5)$.

7. Let $A_1, \dots, A_r$ be non-zero vectors which are mutually perpendicular, in other words $A_i \cdot A_j = 0$ if $i \neq j$. Let $c_1, \dots, c_r$ be numbers such that

$$c_1 A_1 + \cdots + c_r A_r = O.$$

   Show that all $c_i = 0$.

8. For any vectors A, B, prove the following relations:
   (a) $\|A + B\|^2 + \|A - B\|^2 = 2\|A\|^2 + 2\|B\|^2$.
   (b) $\|A + B\|^2 = \|A\|^2 + \|B\|^2 + 2A \cdot B$.
   (c) $\|A + B\|^2 - \|A - B\|^2 = 4A \cdot B$.
   Interpret (a) as a "parallelogram law".

9. Show that if θ is the angle between A and B, then

$$\|A - B\|^2 = \|A\|^2 + \|B\|^2 - 2\|A\| \, \|B\| \cos \theta.$$

10. Let A, B, C be three non-zero vectors. If $A \cdot B = A \cdot C$, show by an example that we do not necessarily have $B = C$.

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I, §5. EXERCISES

1. Find a parametric representation for the line passing through the following pairs of points.
   (a) $P_1 = (1, 3, -1)$ and $P_2 = (-4, 1, 2)$
   (b) $P_1 = (-1, 5, 3)$ and $P_2 = (-2, 4, 7)$

Find a parametric representation for the line passing through the following points.

2. $(1, 1, -1)$ and $(-2, 1, 3)$
3. $(-1, 5, 2)$ and $(3, -4, 1)$
4. Let $P = (1, 3, -1)$ and $Q = (-4, 5, 2)$. Determine the coordinates of the following points:
   (a) The midpoint of the line segment between P and Q.
   (b) The two points on this line segment lying one-third and two-thirds of the way from P to Q.
   (c) The point lying one-fifth of the way from P to Q.
   (d) The point lying two-fifths of the way from P to Q.
5. If P, Q are two arbitrary points in n-space, give the general formula for the midpoint of the line segment between P and Q.

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1. Show that the lines $2x + 3y = 1$ and $5x - 5y = 7$ are not perpendicular.
2. Let $y = mx + b$ and $y = m'x + c$ be the equations of two lines in the plane. Write down vectors perpendicular to these lines. Show that these vectors are perpendicular to each other if and only if $mm' = -1$.

Find the equation of the line in 2-space, perpendicular to N and passing through P, for the following values of N and P.

3. $N = (1, -1)$, $P = (-5, 3)$
4. $N = (-5, 4)$, $P = (3, 2)$
5. Show that the lines

$$3x - 5y = 1, \qquad 2x + 3y = 5$$

are not perpendicular.

6. Which of the following pairs of lines are perpendicular?
   (a) $3x - 5y = 1$ and $2x + y = 2$
   (b) $2x + 7y = 1$ and $x - y = 5$
   (c) $3x - 5y = 1$ and $5x + 3y = 7$
   (d) $-x + y = 2$ and $x + y = 9$

7. Find the equation of the plane perpendicular to the given vector N and passing through the given point P.
   (a) $N = (1, -1, 3)$, $P = (4, 2, -1)$
   (b) $N = (-3, -2, 4)$, $P = (2, \pi, -5)$
   (c) $N = (-1, 0, 5)$, $P = (2, 3, 7)$

8. Find the equation of the plane passing through the following three points.
   (a) $(2, 1, 1)$, $(3, -1, 1)$, $(4, 1, -1)$
   (b) $(-2, 3, -1)$, $(2, 2, 3)$, $(-4, -1, 1)$
   (c) $(-5, -1, 2)$, $(1, 2, -1)$, $(3, -1, 2)$

9. Find a vector perpendicular to $(1, 2, -3)$ and $(2, -1, 3)$, and another vector perpendicular to $(-1, 3, 2)$ and $(2, 1, 1)$.

10. Find a vector parallel to the line of intersection of the two planes

$$2x - y + z = 1, \qquad 3x + y + z = 2.$$

11. Same question for the planes,

$$2x + y + 5z = 2, \qquad 3x - 2y + z = 3.$$

12. Find a parametric representation for the line of intersection of the planes of Exercises 10 and 11.

13. Find the cosine of the angle between the following planes:
   (a) $x + y + z = 1$, $x - y - z = 5$
   (b) $2x + 3y - z = 2$, $x - y + z = 1$
   (c) $x + 2y - z = 1$, $-x + 3y + z = 2$
   (d) $2x + y + z = 3$, $-x - y + z = \pi$

14. (a) Let $P = (1, 3, 5)$ and $A = (-2, 1, 1)$. Find the intersection of the line through P in the direction of A, and the plane $2x + 3y - z = 1$.
   (b) Let $P = (1, 2, -1)$. Find the point of intersection of the plane

$$3x - 4y + z = 2,$$

   with the line through P, perpendicular to that plane.

15. Let $Q = (1, -1, 2)$, $P = (1, 3, -2)$, and $N = (1, 2, 2)$. Find the point of the intersection of the line through P in the direction of N, and the plane through Q perpendicular to N.

16. Find the distance between the indicated point and plane.
   (a) $(1, 1, 2)$ and $3x + y - 5z = 2$
   (b) $(-1, 3, 2)$ and $2x - 4y + z = 1$
   (c) $(3, -2, 1)$ and the yz-plane
   (d) $(-3, -2, 1)$ and the yz-plane

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17. Draw the triangle with vertices $A = (1, 1)$, $B = (2, 3)$, and $C = (3, -1)$. Draw the point P such that $\overrightarrow{AP} \perp \overrightarrow{BC}$ and P belongs to the line passing through the points B and C.
    (a) Find the cosine of the angle of the triangle whose vertex is at A.
    (b) What are the coordinates of P?

18. (a) Find the equation of the plane M passing through the point $P = (1, 1, 1)$ and perpendicular to the vector $\overrightarrow{ON}$, where $N = (1, 2, 0)$.
    (b) Find a parametric representation of the line L passing through

$$Q = (1, 4, 0)$$

and perpendicular to the plane M.
    (c) What is the distance from Q to the plane M?

19. Find the cosine of the angle between the planes

$$2x + 4y - z = 5 \qquad \text{and} \qquad x - 3y + 2z = 0.$$

7. Compute $E_1 \times (E_1 \times E_2)$ and $(E_1 \times E_1) \times E_2$. Are these vectors equal to each other?

8. Carry out the proofs of **CP 1** through **CP 4**.

9. Compute the area of the parallelogram spanned by the following vectors.
    (a) $A = (3, -2, 4)$ and $B = (5, 1, 1)$
    (b) $A = (3, 1, 2)$ and $B = (-1, 2, 4)$
    (c) $A = (4, -2, 5)$ and $B = (3, 1, -1)$
    (d) $A = (-2, 1, 3)$ and $B = (2, -3, 4)$

Do the next exercises after you have read Chapter II, §1.

10. Using coordinates, prove that if $X(t)$ and $Y(t)$ are two differentiable curves (defined for the same values of t), then

$$\frac{d[X(t) \times Y(t)]}{dt} = X(t) \times \frac{dY(t)}{dt} + \frac{dX(t)}{dt} \times Y(t).$$

11. Show (using only Exercise 10) that

$$\frac{d}{dt}[X(t) \times X'(t)] = X(t) \times X''(t).$$

12. Let $Y(t) = X(t) \cdot (X'(t) \times X''(t))$. Show that

$$Y'(t) = X(t) \cdot (X'(t) \times X'''(t)).$$