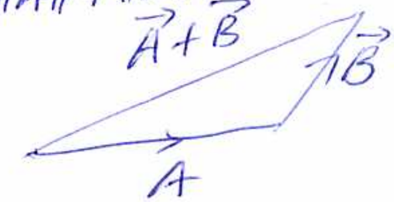
<u>Norm $\|\vec{A}\|$ of a vector $\vec{A}$</u> is $\|\vec{A}\| = \sqrt{a_1^2+a_2^2+a_3^2}$
(length)

$$\|\vec{A}\|^2 = \vec{A}\cdot\vec{A}$$

<u>Theorem</u> If $\vec{A}$ + $\vec{B}$ are $\perp^r$, then $\|\vec{A}+\vec{B}\|^2 = \|\vec{A}\|^2 + \|\vec{B}\|^2$

<u>Theorem:</u> $\|\vec{A}+\vec{B}\| \le \|\vec{A}\| + \|\vec{B}\|$

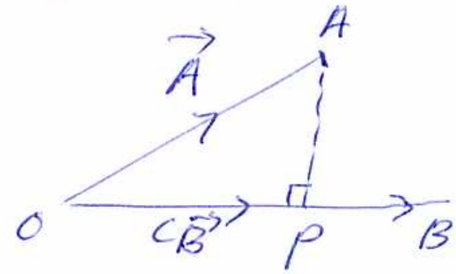


(Dot) <u>Scalar product of 2 vectors</u>

$$\vec{A}\cdot\vec{B} = (a_1, a_2, a_3)\cdot(b_1, b_2, b_3)$$
$$= a_1b_1 + a_2b_2 + a_3b_3$$

(Projection) <u>Component of vector $\vec{A}$</u> along the vector $\vec{B}$ is

$$c\vec{B} = \frac{A\cdot B}{B\cdot B}\vec{B}$$



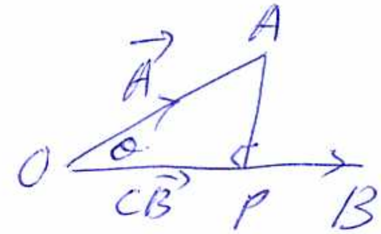
Let $\vec{OP} = c\vec{B}$

Then $\vec{PA} \perp \vec{OP}$:

$$\vec{PA}\cdot\vec{OP} = 0$$
$$(\vec{A}-\vec{P})\cdot cB = 0$$
$$(A-cB)\cdot cB = 0$$
$$(A-cB)\cdot B = 0$$

$$c\, B\cdot B = A\cdot B$$
$$c = \frac{A\cdot B}{B\cdot B}$$

Angle θ between $\vec{A}$ + $\vec{B}$.

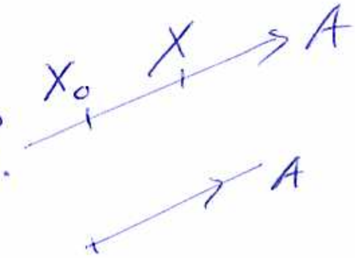


$$\cos\theta = \frac{\|c\vec{B}\|}{\|A\|} = \frac{c\|\vec{B}\|}{\|A\|}$$

$$= \frac{A\cdot B}{B\cdot B}\frac{\|B\|}{\|A\|} = \frac{A\cdot B}{\|B\|\,\|A\|}$$

$$\therefore\ A\cdot B = \|A\|\|B\|\cos\theta$$

Parametric Eq of A line passing through X_0 + $\|$ to vector $\vec{A}$.



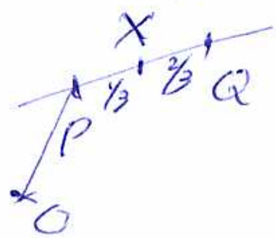
Let X be any pt on line.

Then $X - X_0 = tA$

$X = X_0 + tA$, t is the parameter, i.e. $t \in \mathbb{R}$.

or $(x,y,z) = (x_0,y_0,z_0) + t(a,b,c)$.

Ex. Find the pt X which is $\frac{1}{3}$ of the way from pt P to pt Q

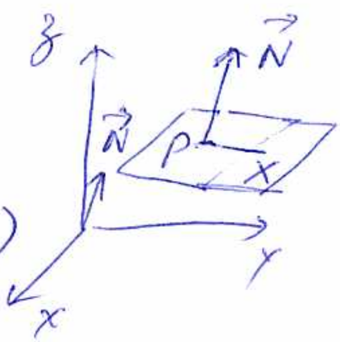


$$\vec{PX} = \frac{1}{3}\vec{PQ}$$
$$\vec{X} - \vec{P} = \frac{1}{3}\vec{PQ}$$
$$X = \vec{P} + \frac{1}{3}(\vec{Q} - \vec{P})$$
$$= \frac{2}{3}\vec{P} + \frac{1}{3}\vec{Q}$$

(note not: $\frac{\vec{P}+\vec{Q}}{3}$)

Eq of Plane (in 3-D)

Containing pt P and $\perp$ vector $\vec{N}$. (normal vector)



Let X be any pt in the plane.

Then $\vec{PX} \perp \vec{N}$.

$\therefore (\vec{X}-\vec{P})\cdot\vec{N} = 0$

$\vec{X}\cdot\vec{N} = \vec{P}\cdot\vec{N}$ $\qquad \vec{N} = (a,b,c)$ say

i.e.

$$\underline{ax+by+cz} = ax_0+by_0+cz_0 = \underline{d}$$

is the Eq of the plane containing pt (x_0, y_0, z_0) + $\perp^r$ $\vec{N}=(a,b,c)$

Ex. Find the eq of plane containing pt $(1,2,3)$ + $\perp^r$ to $\vec{N}=(1,3,-2)$ is

$$x+3y-2z = 1+6-6 = 1.$$

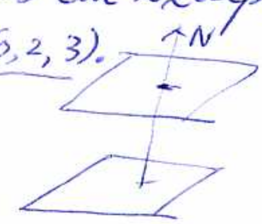
Ex. If $d=0$, i.e. the plane

$ax+by+cz=0$ passes through origin

Sketch this plane. ($x+y+2z=0$, as an example).

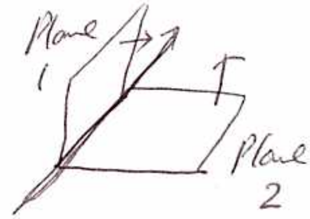
Ex. Find the plane passing thru $A(1,1,2)$, $B(1,0,-1)$ + $C(0,2,3)$.

// planes: have the same normal $\vec{N}$.



2 planes are either //
or they intersect.

Intersection of 2 Planes



When 2 lines in xy-plane intersect, they intersect at a pt.;
when 2 planes (in 3-D) intersect, they have a common line of intersection

Plane 1: $x - y + z = 1$ —①

" 2: $2x + y - 3z = 2$ —②

① + ②:

$$3x \quad - 2z = 3$$

Let $z = t$

$$\therefore x = \frac{1}{3}(3 + 2z) = 1 + \frac{2}{3}t$$

$$y = x + z - 1 = 1 + \frac{2}{3}t + t - 1 = \frac{5}{3}t$$

∴ The line of intersection is

$$x = 1 + \frac{2}{3}t, \quad y = \frac{5}{3}t, \quad z = t.$$

or $(x, y, z) = (1, 0, 0) + t\left(\frac{2}{3}, \frac{5}{3}, 1\right)$

$= (1, 0, 0) + s(2, 5, 3)$

$t, s \in \mathbb{R}$

Can also: Let $x = t$

$$2z = -3 + 3x \quad \therefore z = \frac{-3}{2} + \frac{3}{2}t$$

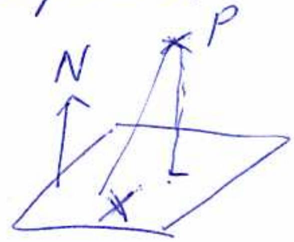
$$y = x + z - 1 = t - \frac{3}{2} + \frac{3}{2}t - 1 = \frac{-5}{2} + \frac{5}{2}t.$$

i.e.

$$x = t$$
$$y = \frac{-5}{2} + \frac{5t}{2}$$
$$z = \frac{-3}{2} + \frac{3}{2}t.$$

Distance (shortest) between a pt P and a plane.

$$= \frac{|(\vec{P}-\vec{X})\cdot\vec{N}|}{\|\vec{N}\|}$$



Ex. Pt. $P(1,2,-1)$ and Plane $x-y+2z=2$. Chose $X=(1,1,1)$ on the plane.

Shortest dist. is $\dfrac{|((1,2,-1)-(1,1,1))\cdot(1,-1,2)|}{\sqrt{1^2+1^2+2^2}}$

(Vector) $= \dfrac{|(0,1,-2)\cdot(1,-1,2)|}{\sqrt{6}} = \dfrac{|-1-4|}{\sqrt{6}} = \dfrac{5}{6}.$

Cross Product of 2 vectors

$$\vec{A}\times\vec{B} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \qquad \begin{aligned} \underset{\sim}{i} &= (1,0,0) \\ \underset{\sim}{j} &= (0,1,0) \\ \underset{\sim}{k} &= (0,0,1) \end{aligned}$$

$$= \underset{\sim}{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \underset{\sim}{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \underset{\sim}{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2,\ -(a_1b_3 - a_3b_1),\ a_1b_2 - a_2b_1)$$

is a vector.

Ex. Find the vector $\perp^r$ to the plane containing 3 pts $A=(1,0,1)$, $B=(0,2,3)$, $C=(-1,-1,0)$. & find the eq of the plane.

Ex. P48 prob. 9

Cross (vector) Product

Right-hand screw

$B \times A = -(A \times B)$

CP 1. $A \times B = -(B \times A)$

CP 2. $(A \times B) \times C = (A \cdot C)B - (B \cdot C)A$

$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

$A \times B$ is $\perp^r$ A

& $\perp^r$ B.

Area of //gram spanned by vectors $\vec{A}$ & $\vec{B}$:

$$\|\vec{A} \times \vec{B}\| = \|\vec{A}\| \, \|\vec{B}\| \sin \theta$$