Acceleration vector is $X''(t)$.

Scalar acceleration is

rate of change of speed $v(t)$. $\frac{dv(t)}{dt} = v'(t)$

Note: The norm of $X''(t)$, $\|X''(t)\|$ is not the same as $v'(t)$.

$$\frac{d}{dt} X^2(t) = \frac{d}{dt} X(t) \cdot X(t)$$

$$= X(t) \cdot X'(t) + X'(t) \cdot X(t)$$

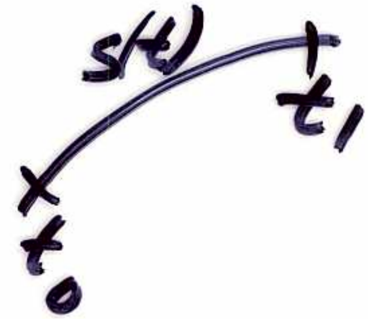
$$= 2X(t) \cdot X'(t).$$

Find the angle between $X(t)$ & $X'(t)$:

$$X(t) = \left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}, 1\right).$$

Length of a Curve (or Arc length)

$s(t)$



$$\frac{ds}{dt} = v(t)$$

$$\therefore\ s = \int_{t_0}^{t_1} v(t)\,dt = \int_{t_0}^{t_1} \|X'(t)\|\,dt$$

$$= \int_{t_0}^{t_1} \sqrt{x'^2 + y'^2}\,dt, \text{ if } X(t) = (x(t), y(t))$$

or

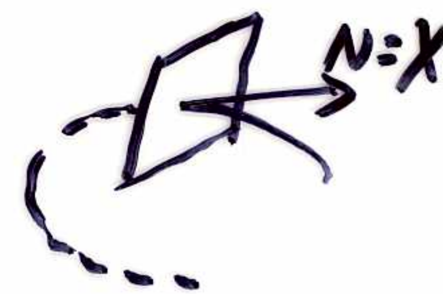
$$* \quad s = \int_{t_0}^{t_1} \sqrt{x'^2 + y'^2 + z'^2}\,dt, \text{ if } X(t) = (x(t), y(t), z(t))$$

Ex. $X(t) = \left(t, t^2, \frac{2}{3}t^3\right)$ from $t = 0$ to $t = 1$.

Find the eq of plane $\perp^r$ to the spiral

$$X(t) = (\cos t, \sin t, t) \text{ at } t = \frac{\pi}{4}.$$

Eq of the plane is



$$X \cdot N = P \cdot N$$

ie

$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y + z = -\frac{1}{2} + \frac{1}{2} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$N = X'\left(\frac{\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)$$

$X'(t)$ is called the velocity vector

Speed is $v(t) = \|X'(t)\|$.

$$\therefore \; v^2(t) = X'(t) \cdot X'(t)$$

or $v^2 = X' \cdot X' = X'^2$ (omitting "t")

If $X(t) = (\cos t, \sin t, t)$, we get

$$v(t) = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}.$$

A 3-D curve is

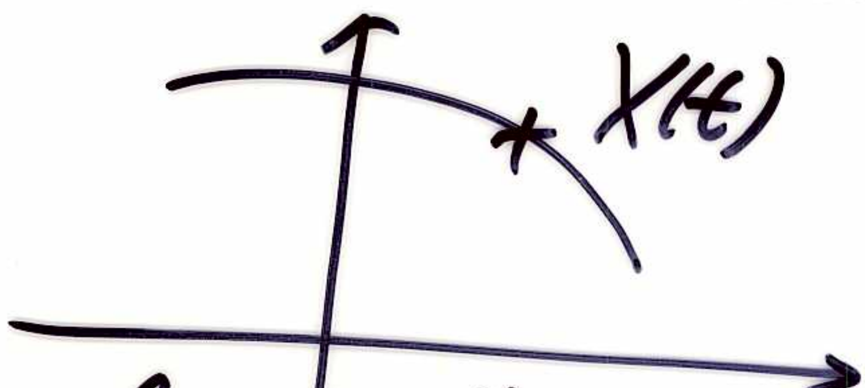
$X(t) = (x(t), y(t), z(t))$, where $t \in \mathbb{R}$

parameter

A 3-D line is a special case of a curve and is of the form:

$$X(t) = X_0 + t\vec{A}, \quad \vec{A} = (a, b, c)$$

vector

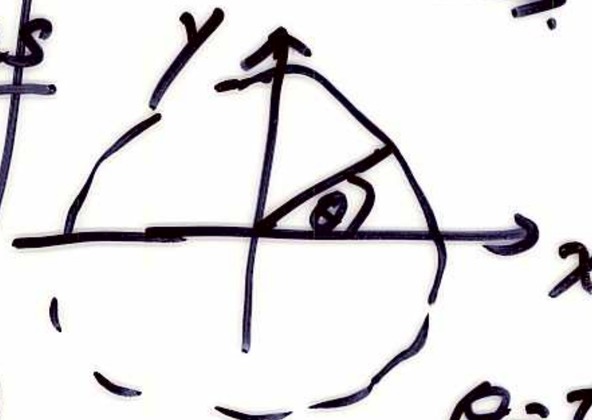


Examples of Curves

(1) 2D curve

$(x(t), y(t))$

$= (\cos t, \sin t)$



$\theta = t$.

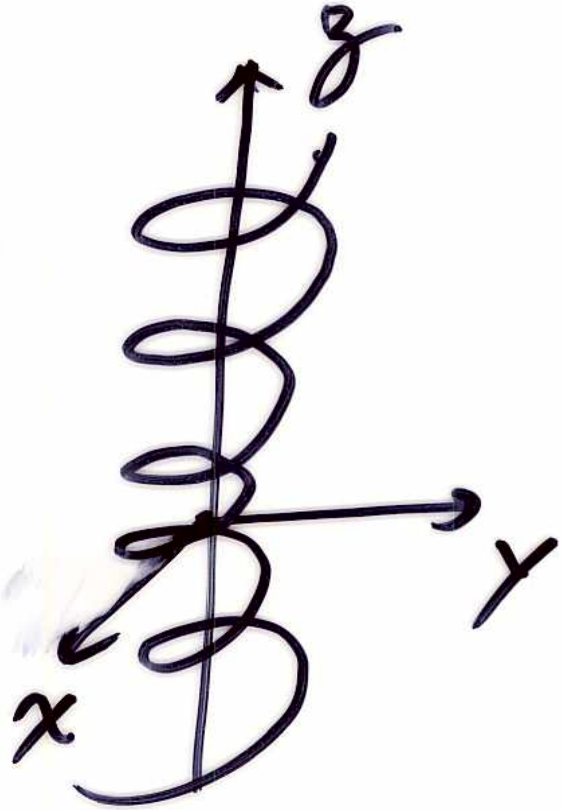
(2) $(x(t), y(t)) = (a\cos t, b\sin t)$

ellipse.

(3) 3-D curve

$X(t) = (\cos t, \sin t, t)$

spiral.



Derivative of a Vector function

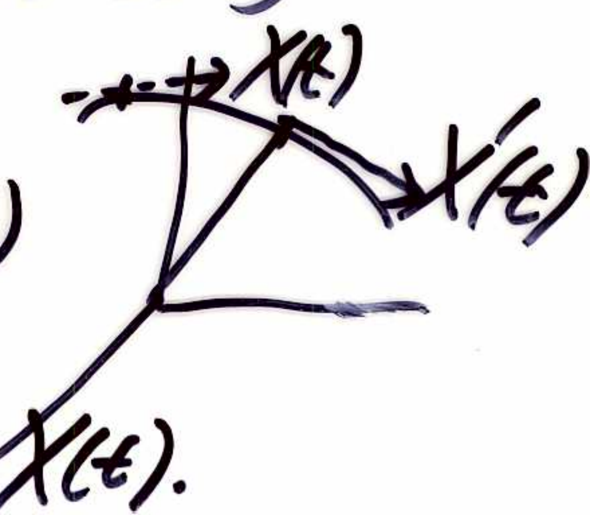
$$X(t) = (x(t), y(t), z(t)).$$

Similar to derivative of a scalar function

$$\therefore \frac{dX(t)}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

$\swarrow$

Vector function $X'(t)$ is tangential to the curve at pt. $X(t)$.



Tangent line to a curve $X(t) = (\cos t, \sin t, t)$ at the pt. $t = \pi/4$.

$$X'(t) = (-\sin t, \cos t, 1)$$

$$X'(\tfrac{\pi}{4}) = (-\sin\tfrac{\pi}{4}, \cos\tfrac{\pi}{4}, 1)$$

$$= \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1\right).$$

$$\text{Pt. } P = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right).$$

$\therefore$ Tangent line at P is

$$X(t) = P + t\,X'(\tfrac{\pi}{4})$$

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) + t\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)$$

or $s(-\sqrt{2}, \sqrt{2}, 2)$