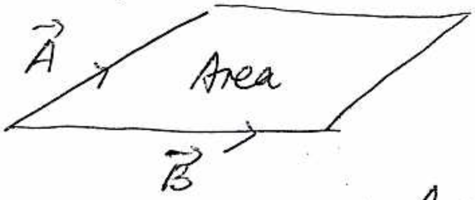
Spring '05

MATH 331 (L20)
Assignment 3 (Solution)

1 (i)



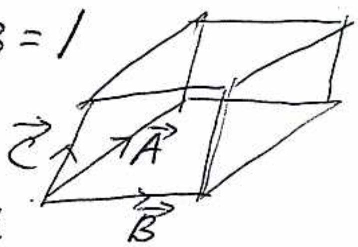
$$\vec{A}\times\vec{B} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{vmatrix} = (-1, 0, 1)$$

$\therefore$ Area of ∥gram $= \|\vec{A}\times\vec{B}\| = \sqrt{1^2+1^2} = \sqrt{2}$. Sq. units

(ii) $(\vec{A}\times\vec{B})\cdot\vec{C}$

$= (-1, 0, 1)\cdot(2, 0, 3) = -2+3 = 1$

$\therefore$ Volume $= |(\vec{A}\times\vec{B})\cdot\vec{C}| = 1$. cubic unit



(iii) $\vec{PQ} = (2, 2, 2)$

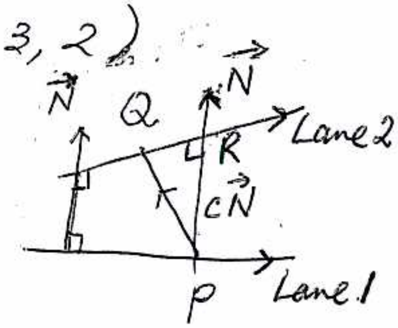
$\vec{PR} = (0, 1, 0)$

$$\therefore \vec{PQ}\times\vec{PR} = \begin{vmatrix} i & j & k \\ 2 & 2 & 2 \\ 0 & 1 & 0 \end{vmatrix} = (-2, 0, 2).$$

$\therefore$ Area of $\triangle PQR = \frac{1}{2}\|\vec{PQ}\times\vec{PR}\| = \frac{1}{2}\sqrt{2^2+2^2} = \sqrt{2}$

2. Find $\vec{N}$, normal to $(-1,-1,2)$ and $(1,-1,1)$

$\therefore \vec{N} = (-1,-1,2)\times(1,-1,1) = (1, 3, 2)$.



Component of $\vec{PQ}$ in the direction of $\vec{N}$.

$$= c\vec{N} = \frac{\vec{PQ}\cdot\vec{N}}{\|\vec{N}\|^2}\vec{N}$$

$$\therefore \text{Dist.} = \|c\vec{N}\| = \frac{|PQ\cdot\vec{N}|}{\|N\|^{\cancel{2}}}\cancel{\|\vec{N}\|} = \frac{|(0,1,-1)\cdot(1,3,2)|}{\sqrt{14}}$$

$$= \frac{|3-2|}{\sqrt{14}} = \frac{1}{\sqrt{14}}.$$

3. (i) Arc length $= \int_0^{1/3}\sqrt{x'^2+y'^2+z'^2}\,dt$

$x' = 3\frac{e^{3t}}{3} = e^{3t}$

$y' = -e^{-3t}$

$z' = \sqrt{2}$

$\therefore x'^2+y'^2+z'^2 = e^{6t}+2+e^{-6t}$

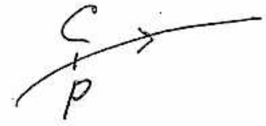
$= (e^{3t}+e^{-3t})^2$

$$= \int_0^{1/3} e^{3t}+e^{-3t}\,dt$$

$$= \frac{1}{3}\left(e^{3t}-e^{-3t}\right)\Big]_0^{1/3}$$

$$= \frac{1}{3}\left(e-e^{-1}+(1-1)\right) = \frac{1}{3}\left(e-\frac{1}{e}\right)$$

3 (ii) $X' = (e^{3t}, -e^{-3t}, \sqrt{2})$



$X'(P) = (e, -e^{-1}, \sqrt{2})$

Eqn tgt line at P is

$$X = P + t(e, -e^{-1}, \sqrt{2}), \quad t \in R.$$

$$= \left(\frac{e}{3}, \frac{e^{-1}}{3}, \frac{\sqrt{2}}{3}\right) + t(e, -e^{-1}, \sqrt{2})$$

4. (i) $X(t) = (x, y, z) = (2t^2, 1-t, 3+t^2)$ & plane $3x-14y+z=0$ intersect at:

$$3(2t^2)-14(1-t)+(3+t^2) = 10.$$

$$7t^2+14t-21 = 0$$

$$7(t+3)(t-1) = 0 \quad \therefore t = -3, 1$$

i.e at pts $(18, 4, 12)$ and $(2, 0, 4)$.

(ii). Use scalar product : $X' \cdot N = X' \cdot (3, -14, 1) = \|X'\|\|N\|\cos\theta$.

At pt P $(18, 4, 12)$, $(4t, -1, 2t)_P \cdot (3, -14, 1)$

$$= (-12, -1, -6) \cdot (3, -14, 1)$$

$$= -28 = \sqrt{144+1+36}\,\sqrt{9+196+1}\,\cos\theta$$

$$\therefore \theta = \cos^{-1}\frac{-28}{\sqrt{181}\,\sqrt{206}}.$$

Similarly for $Q(2, 0, 4)$.