MATHEMATICS 331 — ASSIGNMENT #1

DUE DATE: FEBRUARY 5, 2004

1. Find the solution of $x' = Ax$ with initial value
$$x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \text{if} \qquad A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}.$$

2. If $\Phi(t)$ is a fundamental solution for $x' = Ax$, Find a fundamental solution $\Psi(t)$ if $\Psi(0) = I$.

3. Show that if λ is a complex eigenvalue of the real matrix A, with corresponding eigenvector c, then the real and imaginary parts
$$\mathrm{Re}(e^{\lambda t}c) \qquad \text{and} \qquad \mathrm{Im}(e^{\lambda t}c)$$
are linearly independent.

4. From Euler's formula we get
$$(\cos\theta + i\sin\theta)^n = (e^{i\theta})^n = e^{in\theta} = \cos n\theta + i\sin n\theta.$$
Write out the real and imaginary parts of this using the binomial theorem and $i^2 = -1$ to get trigonometric identities for $\cos n\theta$ and $\sin n\theta$ in terms of $\cos\theta$ and $\sin\theta$. Check your answer for the case $n = 4$.

5. Find the eigenvalues and corresponding eigenvectors of
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 2 \end{pmatrix}.$$
Explain why this makes it difficult to find the exponential e^{tA} with the techniques developed so far in class.

6. (Extra for experts). By analogy with what we did in class for 2×2 matrices see if you can find e^{tA} if
$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$