

1. Compute the exponential  $e^{tA}$  of the matrix  $A$  if

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

2. Show that the matrices  $A$  and  $B$  are similar if

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & 1 \\ -2 & 8 \end{pmatrix}.$$

Hint: Try to solve  $TA = BT$  for an unknown  $T$ .

3. If  $A$  is the matrix

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 0 \\ 2 & -2 & 3 \end{pmatrix},$$

- (a) Find a fundamental matrix  $\Phi(t)$  for the differential equation  $x' = Ax$ .  
(b) Find the solution of  $x' = Ax$  that satisfies

$$x(0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

4. Repeat the previous exercise, only this time for the matrix

$$B = \begin{pmatrix} -1 & -1 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$