1. If F is a differentiable function of two variables, and
$$u = F(s^2 - t^2, t^2 - s^2),$$
show that
$$t\frac{\partial u}{\partial s} + s\frac{\partial u}{\partial t} = 0.$$

2. If
$$u = x^3 F\left(\frac{y}{x}, \frac{z}{x}\right),$$
show that
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u.$$

3. Consider the four variables V, S, r, h connected by the two equations
$$V = \pi r^2 h, \qquad S = 2\pi r h + 2\pi r^2.$$
These formulae arise if we consider a right circular cylinder of height h, radius of base r, volume V, and total surface area S. Of the four variables, just two are independent. Ordinarily we most naturally think of r and h as independent, but other choices are legitimate. We may choose r and S as independent. Then
$$h = \frac{S}{2\pi r} - r, \qquad V = \frac{1}{2}rS - \pi r^3.$$
In view of what has been said, it is evident that the notation $\partial V/\partial r$ is ambiguous, for if we calculate from the first pair of equations we have $\partial V/\partial r = 2\pi r h$, while from the second pair $\partial V/\partial r = \frac{1}{2}S - 3\pi r^2$, and these two results are not in agreement. A customary resolution is to employ subscripts. According to this practice,
$$\left(\frac{\partial V}{\partial r}\right)_h$$
indicates that we are regarding V as a function of the two independent variables r, h with h held constant in the differentiation.

(a) Compute the derivatives

$$\left(\frac{\partial V}{\partial h}\right)_r, \quad \left(\frac{\partial V}{\partial S}\right)_r, \quad \left(\frac{\partial S}{\partial h}\right)_r.$$

(b) Also compute the derivatives

$$\left(\frac{\partial S}{\partial V}\right)_h, \quad \left(\frac{\partial h}{\partial V}\right)_r, \quad \left(\frac{\partial r}{\partial S}\right)_h.$$