DUE DATE: FEBRUARY 26, 2004

1. Recall the old wind chill formula

$$W(T, V) = 91 + (0.44 + 0.325\sqrt{V} - 0.023V)(T - 91),$$

as well as the revised (November 2001) one

$$W(T, V) = 35.74 + 0.6215T - 35.75V^{0.16} + 0.4275TV^{0.16}.$$

Also recall that the celsius temperature C and the fahrenheit temperature F are related by

$$9C - 5F + 160 = 0.$$

Simultaneously plot, for $C = -10$, the graphs of the old and new wind chill factors, with the wind speed on the abcissa and the wind chill on the ordinate. Give a commentary on the difference. You will probably want to use a computer to draw an accurate plot, and may wish to use a free graphing program, such as gnuplot.

2. Describe all possible level sets for the function

$$f(x, y) = 1 - (x^2 + y^2 - 1)^2.$$

3. By finding two different paths to the origin, show that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$$

does not exist.

4. Let A be the matrix

$$A = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix},$$

and $F(x_1, x_2, x_3, x_4) = A^{-1}$.

   (a) What is the domain of F?

   (b) Compute the partial derivatives of F.

5. Show that $u(x,t) = 2k^2\operatorname{sech}^2(k(x - 4k^2t))$ is a solution (the famous soliton) of the KdV equation

$$u_t + 6uu_x + u_{xxx} = 0.$$

6. (Extra for experts). A function $f(x,y)$ is called homogeneous of degree k if

$$f(ax, ay) = a^k f(x,y).$$

   (a) Find the most general polynomial of the two variables x and y that is homogeneous of degree three.

   (b) Prove *Euler's theorem* on homogeneous functions:

$$xD_1 f(x,y) + yD_2 f(x,y) = kf(x,y).$$