1. Let A and B be the matrices

$$A = \begin{pmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ 0 & 1 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 & 3 \\ -1 & 4 & 2 \\ 6 & 1 & 0 \end{pmatrix}$$

   Compute $A+B$, AB, BA, and A^{-1} if it exists.

2. Show that the product of two diagonal matrices is diagonal.

3. Find $A'(t)$ and $\int_0^t A(s)ds$ if

$$A(t) = \begin{pmatrix} e^t & e^t \cos t \\ e^t & e^t \sin t \end{pmatrix}$$

4. Define an operator T on the space of continuous functions on $[0,1]$ by integration:

$$(Tf)(t) = \int_0^t f(s)\, ds$$

   Show that T is a linear operator.

5. Consider the differential equation $x' = Ax$ where

$$A = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$$

   Check that

$$\begin{aligned} x_1(t) &= ae^t - 2be^{2t} \\ x_2(t) &= be^{2t} \end{aligned}$$

   where a and b are arbitrary constants, solves the differential equation.

   (a) From this, find a fundamental matrix Φ for the system, and hence solve the initial value problem for when $(x_1(0) = 1, x_2(0) = 0)$ and also for when $(x_1(0) = 1, x_2(0) = 1)$.

   (b) Compute the trace of A and the determinant of the fundamental matrix Φ. Letting $z(t) = |\Phi(t)|$, show that

$$\frac{z'(t)}{z(t)} = \operatorname{tr}(A)$$