

1. What is the natural domain of

$$f(x, y) = \arcsin \left(\frac{1}{\sqrt{3 - x^2 - 2y^2}} \right)$$

2. Consider the function defined for $(x, y) \neq (0, 0)$ by the formula

$$f(x, y) = \frac{xy}{x^2 + y^2}.$$

A reasonable thing to attempt is to try to make this function defined everywhere by taking the limit as we go to the origin. To see the difficulty, compute the limit along the straight lines $y = kx$ of slope k

$$\lim_{x \rightarrow 0} f(x, kx).$$

3. Find the coordinates of all points on the surface with equation

$$z = x^4 - 4xy^3 + 6y^2 - 2$$

where the surface has a horizontal tangent plane.

4. Compute the first partial derivatives of $f(x, y) = \sqrt{x^2 - xy}$. Where are these derivatives defined?
5. Find D_1g , D_2g , $D_1(D_1g)$, $D_1(D_2g)$, $D_2(D_1g)$ and $D_2(D_2g)$ of

$$g(x, y) = e^{xy^2}.$$

6. Show that $f(x, y) = \log \sqrt{x^2 + y^2}$ satisfies the partial differential equation

$$f_{xx} + f_{yy} = 0.$$

7. If $f(x) = (x^2, \pi(x-1)/2, 2x-3)$, $g(x, y, z) = x^2 \cos(yz)$, and $h = g \circ f$, find the derivative

$$\frac{dh}{dx}$$

when $x = 2$.