FINAL HANDOUT
MATH 349 .

1. Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(3x-1)^n}{3^{3n}\sqrt{n}}$.

2. Find the Taylor series of $f(x) = xe^x$ around the centre $c = -1$.
   For which x is the represenation valid?

3. For the curve c given as the intersection of two surfaces
   $c = \{z = x^2 + y^2\} \cap \{6x - 2y - z = 1\}$
   (a) find a parametrization;
   (b) find an equation of the tangent line at $P\,(0,-1,1)$;
   (c) set up the integral for evaluating arclength between the points P and $R\,(3,2,13)$ .Do not evaluate.

4. Find an equation of the tangent plane to the surface
   $z = x^{yx^2}$ at the point $x = 1, y = -1$.

5. For the function f defined as follows
   $f(x,y) = \dfrac{x^2}{x^2 + 3y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$
   (a) find the gradient ∇f at the origin;
   (b) is f continuous at the origin? Explain.

6. Find the domain of definition,range and level curves ($c = 0, 1, -1$)
   for $z = \ln \frac{x}{y}$.

7. Find the directional derivative of $f\,(x,y,z) = e^{-y}\,(x^2 + \cos z)$
   at the point $A\,(2,0,\pi)$ in the direction towards the point $B\,(-1,-1,0)$.

8. Given $f\,(2,-1) = 11$, $f_x\;(2,-1) = -7$,$f_y\,(2,-1) = 5$, $f(2,-2) = 9$,
   $f_x(2,-2) = 3$, $f_y(2,-2) = -2$ and $\mathbf{g}\,(u,v) = \left(uv^2, \frac{u}{v}\right)$
   find $\dfrac{\partial F}{\partial u}$ at the point $(2,-1)$ if $F = f \circ \mathbf{g}$
   i.e. $F(u,v) = f(x,y)$ where $x = uv^2, y = \frac{u}{v}$.

9. The temperature T at points (x,y) of the $xy-$ plane is given by
   $T\,(x,y) = y^2 - 3x^2$
   (a) Draw (find) the isotherms (level curves) $T = 0, 3, -6$.

(b) In what direction should an ant siting at $(-1,2)$ move to cool off as quickly as possible?

10. Show that the systems of equations

$$\begin{aligned} xy^2 - z + u^2 &= 3 \\ x^3 z + 2y - u &= 0 \\ xu + y - xyz &= 3 \end{aligned}$$

can be solved for x, y, z as functions of u around the point $(x, y, z, u) = (1, 1, -1, 1)$.

11. Find the Taylor polynomial $T_2(x, y)$ for the function $f(x, y) = \ln(y \cos x)$ around the center $(0, 1)$.

12. Show that the equation $\arcsin(zy) + z^3 x + x^2 y + 8 = 0$ can be solved for z as a function of x, y around he point $P(-1, 0, 2)$ then find $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$ at that point..