MATH 349
Midterm Handout

1. Determine if the indicated sequence is bounded, monotonic, and convergent

   a) $a_n = \dfrac{\ell n(n+3)}{n+3}$ b) $b_n = \dfrac{n^n}{n!}$.

2. Determine whether the indicated series is absolutely convergent, conditionally convergent or divergent.(a)$\sum_{k=1}^{\infty} \dfrac{\arctan k}{1+k^2}$

   (b) $\sum_{n=1}^{\infty} (-1)^n \left(\dfrac{1}{\sqrt{n}} - \dfrac{1}{\sqrt{n+1}} \right)$ (c)$\sum_{k=1}^{\infty} k^2 e^{-k}$ (d) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{(n^2-2n)\sqrt{n}}$

3. Find the interval of convergence if

   (a)$\sum_{k=1}^{\infty} \dfrac{2^k}{\sqrt{k}} (x-1)^k$ b) $\sum_{n=1}^{\infty} \dfrac{n!}{n^n} x^n$ (only abs.convergence).

4. Find the sum of $(a) \sum_{k=1}^{\infty} \dfrac{(\ell n\, 2)^k}{k!}$ $(b).\sum_{n=3}^{\infty} \dfrac{(-1)^n}{2^n(n+1)}$.

5. (a) Is the sequence $a_n = \dfrac{2-(-1)^n}{n^2-2n}$, $n \geq 3$ bounded, alternating or convergent?

   (b) Is the sequence $c_n = \dfrac{3^n}{3^n - 2^n}$ convergent ?Is the series $\sum_{n=1}^{\infty} c_n$ convergent ?

6. Find the Taylor series for $f(x) = \dfrac{1}{(x+3)x}$ around the center $x_0 = -1$,

   particularly the coefficient a_6.

   For what values of x is the representation valid?(Hint: Use partial fractions)

7. Find Taylor polynomial of degree 3 for $f(x) = \ln \dfrac{x-1}{x}$ around the centre $x_0 = 2$.

8. Find a parametrization of the curve c given as the intersection of the cone $\{ z = \sqrt{2x^2+2y^2} \}$ and the plane $\{z + x = 1\}$.

9. For the curve c given by $\mathbf{r}(t) = (2t, t^2, \ln t)\,, t > 0$ find

   (a) an equation of the tangent line at $P\,(2,1,0)$;

   (b) the arclength of c between P and $R\,(2e, e^2, 1)$.

10. For the curve c given by $\mathbf{r}\,(t) = (t \sin t, t \cos t, 2t)$

    (a) find an equation of the tangent line to c at the origin ;

    (b) find the arclength between the origin and the point $A\left(\frac{\pi}{2}, 0, \pi\right)$.

11. Find a parametrization of the curve c given as the intersection of two surfaces $c = \{x^2 + y^2 = 2z\} \cap \{3x - 4y - z = 0\}$.