

Give complete, neat, well-reasoned answers to the following questions.

1. (a) Give a precise definition of the limit of a sequence  $\{a_n\}$ . [5]
- (b) Use this definition to show  $\lim_{n \rightarrow \infty} 1/(n^2 + 1) = 0$ . [5]

2. Let the sequence  $\{a_n\}$  be given by

$$a_n = \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$

Determine the convergence or divergence of the the series

$$\sum_{n=1}^{\infty} a_n$$

Clearly state any results you are using in the determination. [5]

3. Let  $f(t)$  be defined by

$$f(t) = \begin{cases} \frac{e^t - 1}{t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

and let  $F(x)$  be given as the integral

$$F(x) = \int_0^x f(t) dt.$$

- (a) Find the power series expansion of  $F(x)$  about the point  $x = 0$ . [6]
  - (b) What is the radius of convergence of this series? [4]
4. Let  $V$  be the vector space of quadratic polynomials  $p(x) = ax^2 + bx + c$ . Let  $T$  be the mapping from  $V$  to  $V$  given by

$$T(p(x)) = \frac{d}{dx}[(x - 1)p(x)].$$

By identifying  $V$  with  $R^3$ , find a matrix for the linear transformation  $T$ . [7]

5. Find an arclength parametrization of the curve determined by the intersection of the two surfaces  $x - y + z = 1$  and  $5x + 2y - 3z = 0$ . [8]