The University of Calgary
Department of Mathematics and Statistics
MATH 349-02
Quiz # 4T

Fall 2006

Name:______________________________**I.D.#:**____________________

1. For $f(x,y) = \ln \frac{y}{x}$
   (a) sketch the domain of f;find the range;
   (b) sketch in the xy-plane the level curves for $c = 0, 1, -1$. [4]

2. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{1-\cos(xy)}{x^4+2y^4}$,if it exists. [3]

3. Find an equation of the tangent plane to $z = \sqrt{3x^2+y^2}$ at $x = -2, y = 2$. [3]

Solutions.

For 1)

for $D = \left\{ \frac{y}{x} > 0 \right\}$ ...both positive or both negative

$e^c = \frac{y}{x}$ $\quad y = e^c x$ $\quad$ lines through the origin without the origin

with positive slopes only

level curves: for $c = 0$ $\quad y = x, x \neq 0$

for $c = -1$ $\quad y = \frac{1}{e}x;$ $\quad$ for $c = 1:$ $\quad y = ex$

the range is $(-\infty, \infty)$.

For 2)

define $g(x,y) = \dfrac{1-\cos(xy)}{x^4+2y^4}$, then for $y \neq 0 \quad g(0,y) = 0$

and for $x \neq 0 \quad g(x,0) = 0$ $\quad$ try a line through the origin $y = x$

for $x \neq 0 \quad g(x,x) = \dfrac{1-\cos(x^2)}{3x^4}$ as $x \to 0$ by L'Hop.Rule twice

$$\lim_{x\to0} \frac{1-\cos(x^2)}{3x^4} = \lim_{x\to0} \frac{\sin(x^2)2x}{12x^3} = \lim_{x\to0} \frac{\sin(x^2)}{6x^2} = \lim_{x\to0} \frac{\cos(x^2)2x}{12x} = \frac{1}{6} \neq 0$$

thus the limit DNE - does not exist.

For 2)

for $x = -2, y = 2$ $\quad z = \sqrt{16} = 4$ $\quad P(-2, 2, 4)$

partials

$f_x = \dfrac{3x}{\sqrt{3x^2+y^2}}$ for $\quad x = -2, y = 2$ $\quad f_x = -\dfrac{3}{2}$

$f_y = \dfrac{y}{\sqrt{3x^2 + y^2}}$ for $\quad x = -2, y = 2 \quad f_y = \dfrac{1}{2}$

thus a normal vector is $\mathbf{n} = (\nabla f, -1) = \left(-\frac{3}{2}, \frac{1}{2}, -1\right)$ or $(3, -1, 2)$ and an equations is $3x - y + 2z = d$ substitute $P\,(-2, 2, 4) \quad -6 - 2 + 8 = 0$, finally

$3x - y + 2z = 0$