The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lecture 02
Quiz # 1R

Fall 2006

Name:______________________________**I.D.#:**__________________

1. Is the sequence $a_n = \dfrac{\ln(n+1)}{n+1}$ ultimately monotonic ,bounded,alternating, convergent?
   EXPLAIN!Find an upper and a lower bound. [4]

2. Find the limit of $b_n = \left(\dfrac{n+2}{n+3}\right)^n$.Is the sequence bounded? [3]

3. Find the limit of $c_n = \dfrac{1+(-1)^n}{\sqrt{n}}$.Is the sequence momotonic? [3]

Solution

For 1)

$a_n = \dfrac{\ln(n+1)}{n+1} > 0$ for $n \geq 1$ $\lim\limits_{n\to\infty} \dfrac{\ln(n+1)}{n+1} = 0$ by L'H.R.$\left(\frac{1}{x+1}\right)$

the sequence is **convergent to** 0 therefore **bounded,** with positive terms

$f'(x) = \left(\dfrac{\ln(x+1)}{x+1}\right)' = \dfrac{1-\ln(x+1)}{(x+1)^2} < 0$ since $\ln(x+1) > 1$ for $x+1 > e$

for $n \geq 2$,$a_1 = \frac{\ln 2}{2} = 0.35, a_2 = \frac{\ln 3}{3} = 0.36$

the sequence is **ult. decreasing** therefore $\frac{\ln 3}{3} = a_2 \geq a_n > 0$

For 2)

$b_n = \left(\dfrac{n+2}{n+3}\right)^n = e^{n\ln\left(\frac{n+2}{n+3}\right)} \to e^L$

where $L = \lim\limits_{x\to\infty} x\left[\ln(x+2) - \ln(x+3)\right] = \lim\limits_{x\to\infty} \frac{\ln(x+2)-\ln(x+3)}{\frac{1}{x}}$

(L'H.R.) $\lim\limits_{x\to\infty}\left[\dfrac{1}{x+2} - \dfrac{1}{x+3}\right] / \left(\frac{-1}{x^2}\right) = \lim\limits_{x\to\infty}\left[\dfrac{-x^2}{(x+2)(x+3)}\right] = -1$

thus $b_n \to \frac{1}{e}$

we can also use $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}\right)^x = e^a$

and $b_n = \left(\dfrac{n+2}{n+3}\right)^n = \left(1-\dfrac{1}{n+3}\right)^n = \left(1-\dfrac{1}{n+3}\right)^{n+3} \cdot \left(1-\dfrac{1}{n+3}\right)^{-2} \to e^{-1}\cdot 1$

the sequence is **convergent** so **bounded**

For 3)

since $c_n = 0$ for n odd and $c_n = \dfrac{2}{\sqrt{n}} > 0$ for n even

the sequence is **not monotonic**

since $\quad 0 \leq c_n \leq \dfrac{2}{\sqrt{n}}$

and $\lim\limits_{n\to\infty} \dfrac{2}{\sqrt{n}} = 0$ $\lim\limits_{n\to\infty} c_n = 0$ by Squ.Theorem