The University of Calgary
Department of Mathematics and Statistics
MATH 349 Lecture 02
Quiz # 1T

Fall 2006

Name:____________________________**I.D.#:**_________________

1. Is the sequence $a_n = ne^{-n}$ ultimately monotonic ,bounded,alternating, convergent? EXPLAIN!Find an upper and a lower bound. [4]

2. Find the limit of $b_n = (n^2+2)^{\frac{1}{n}}$.Is the sequence bounded? [3]

3. Find the limit of $c_n = \dfrac{\cos n}{n^2}$.Is the sequence monotonic? [3]

Solution

For 1)

$a_n = ne^{-n} = \dfrac{n}{e^n} > 0 \lim\limits_{x\to\infty} \dfrac{n}{e^n} = 0$ by L'H.R.$\left(\frac{1}{e^x}\right)$

the sequence is **convergent to** 0 therefore **bounded,** with positive terms

$\dfrac{a_{n+1}}{a_n} = \dfrac{(n+1)}{e^{n+1}} \cdot \dfrac{e^n}{n} = \dfrac{n+1}{ne} < 1$ for any $n \geq 1$ since $1 < n(e-1)$

OR $f'(x) = (xe^{-x})' = e^{-x} - xe^{-x} = e^{-x}(1-x) \leq 0$

the sequence is **decreasing** therefore $\frac{1}{e} = a_1 \geq a_n > 0$

For 2)

$b_n = (n^2+2)^{\frac{1}{n}} = e^{\frac{1}{n}\ln(n^2+2)} \to 1$ since

$$\lim_{x\to\infty} \frac{\ln(x^2+2)}{x} = (\text{L'H.R.twice}) \lim_{x\to\infty} \frac{2x}{x^2+2} = \lim_{x\to\infty} \frac{2}{2x} = 0$$

the sequence is **convergent** so **bounded**

For 3)

since $\cos n$ is sometimes negative sometimes positive

$c_n = \dfrac{\cos n}{n^2}$ is **not monotonic**

since $\quad -1 \leq \cos n \leq 1 \qquad -\dfrac{1}{n^2} \leq \dfrac{\cos n}{n^2} \leq \dfrac{1}{n^2}$

and $\lim\limits_{n\to\infty} \dfrac{\pm 1}{n^2} = 0 \ \lim\limits_{n\to\infty} c_n = 0$ by Squ.Theorem