MATH 349-lec02
Quiz # 2R   Fall 2006

Name:______________________________**I.D.#**

1. Find the sum of the series $\sum_{n=2}^{\infty} \frac{(-1)^n\, 4^{\frac{n}{2}}}{e^n}$. [3]

2. Is the series $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$ convergent or divergent? Explain. [4]

3. Is the series $\sum_{n=1}^{\infty} \frac{n^2 2^n}{(n+1)!}$ convergent or divergent? Explain. [3]

Solution

For1)

$\sum_{n=2}^{\infty} \frac{(-1)^n\, 4^{\frac{n}{2}}}{e^n} = \sum_{n=2}^{\infty} \left(\frac{-2}{e}\right)^n$ is a geom.series with $0 > r = \frac{-2}{e} > -1$

thus by using $\sum_{n=N}^{\infty} r^n = \frac{r^N}{1-r}$ $\qquad s = \frac{\left(\frac{-2}{e}\right)^2}{1+\frac{2}{e}} = \frac{4}{e^2+2e}$.

For2)

the series is convergent

by Comparison test $0 < \frac{\arctan n}{n^2+1} < \frac{\frac{\pi}{2}}{n^2+1} < \frac{const}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2} < \infty$

OR by Integral test

$$\int_1^{\infty} \frac{\arctan x}{x^2+1} dx = \left(\text{by subst.} u = \arctan x, du = \frac{dx}{x^2+1}\right) = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} u du = \frac{1}{2}\left[u^2\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = const$$

we have to show that $f(x) = \frac{\arctan x}{x^2+1}$ is cont.,pos.,decr.

$f'(x) = \left(\frac{\arctan x}{x^2+1}\right)' = \frac{1-2x\arctan x}{(x^2+1)^2} < 0$ for sure for big x

in detail $\qquad \arctan x > \frac{1}{2x}$ for $x \geq 1$, see the graphs

For 3)

the series is convergent by Ratio test

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2 2^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{n^2 2^n} = \left(\frac{n+1}{n}\right)^2 \frac{2}{n+2} \to 1 \cdot 0 = 0 < 1$$