MATH 349 lec02
Quiz # 2T Fall 2006

Name:____________________________**I.D.#**

1. Find the sum of the series $\sum_{n=3}^{\infty} \frac{\pi^n (-1)^n}{2^{2n}}$. [3]

2. Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{e^{n^2}}$ convergent or divergent? Explain. [4]

3. Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2+\sin n)^2}{n}$ convergent or divergent? Explain. [3]

Solution

For1)

$\sum_{n=3}^{\infty} \frac{\pi^n (-1)^n}{2^{2n}} = \sum_{n=3}^{\infty} \left(\frac{-\pi}{4}\right)^n$ is a geom.series with $0 > r = \frac{-\pi}{4} > -1$

thus by using $\sum_{n=N}^{\infty} r^n = \frac{r^N}{1-r}$ $\qquad s = \frac{\left(\frac{-\pi}{4}\right)^3}{1+\frac{\pi}{4}} = \frac{-\pi^3}{64+16\pi}$.

For2)

the series is convergent by Ratio test

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^3}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n^3} = \left(\frac{n+1}{n}\right)^3 \cdot \frac{e^{n^2}}{e^{n^2+2n+1}} = \left(\frac{n+1}{n}\right)^3 \cdot \frac{1}{e^{2n+1}} \to 1 \cdot \tfrac{1}{\infty} = 0 < 1$$

Or by Integral test

$$\int_1^{\infty} x^3 e^{-x^2}\, dx = (\text{ by subst.} u = x^2) = \frac{1}{2}\int_1^{\infty} u e^{-u} du = (\text{ by parts}) =$$

$$= \frac{1}{2}\left[-u e^{-u} - e^{-u}\right]_1^{\infty} = const \text{ since the limit } e^{-\infty} = 0$$

we have to show that $f(x) = x^3 e^{-x^2}$ is cont.,pos.,decr.

$f'(x) = 3x^2 e^{-x^2} + x^3 e^{-x^2}(-2x) = x^2 e^{-x^2}(3-2x^2) < 0$ for $x \geq 2$

For 3)

the series is divergent by Comparison test:

$-1 \leq \sin n \leq 1 \qquad 1 \leq (2+\sin n)^2 \leq 9$

$\displaystyle\sum_{n=1}^{\infty} \frac{(2+\sin n)^2}{n} \geq \sum_{n=1}^{\infty} \frac{1}{n}$ which is harmonic series