The University of Calgary
Department of Mathematics and Statistics
MATH 349-02 Quiz # 3R Fall 2006

Name:____________________________**I.D.#:**__________________

1. Is the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{3n+8}$ absolutely or conditionally convergent or divergent? [3]

2. Find the centre,radius and interval of convergence of power series $\sum_{n=1}^{\infty}\frac{(4x-1)^n}{n2^n}$ [4]

3. Express $f(x)=\frac{1}{x^2}$ in powers of $(x+2)$. On what interval is the representation valid? [3]

Solutions

For1)

first $\sum_{n=1}^{\infty}\left|(-1)^n\frac{\sqrt{n}}{3n+8}\right|=\sum_{n=1}^{\infty}\frac{\sqrt{n}}{3n+8}$

$$\frac{\sqrt{n}}{3n+8}\approx\frac{\sqrt{n}}{3n}=\frac{1}{3\sqrt{n}} \text{ since } \lim_{n\to\infty}\frac{\frac{\sqrt{n}}{3n+8}}{\frac{1}{3\sqrt{n}}}=\lim_{n\to\infty}\frac{3\sqrt{n}\sqrt{n}}{3n+8}=\lim_{n\to\infty}\frac{3}{3+\frac{8}{n}}=1\neq 0$$

and $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}=\infty$ so by Limit Comp.Test $\sum_{n=1}^{\infty}\left|(-1)^n\frac{\sqrt{n}}{3n+8}\right|=\infty$

it is **cond. convergent** by Alternating Test:

we have to show that $a_n=\frac{\sqrt{n}}{3n+8}$ is decr. and convergent to 0

$$\lim_{n\to\infty}\frac{\sqrt{n}}{3n+8}=\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n}}}{3+\frac{8}{n}}=\frac{0}{3}=0$$

now $f'(x)=\left(\frac{\sqrt{x}}{3x+8}\right)'=\frac{\frac{1}{2\sqrt{x}}(3x+8)-\sqrt{x}\cdot 3}{(3x+8)^2}=\frac{(3x+8)-6x}{2\sqrt{x}(3x+8)^2}=\frac{8-3x}{2\sqrt{x}(3x+8)^2}$

and $f'(x)<0$ for $x>3$ thus the sequence is ult.decr.

For 2)

the centre is $c=\frac{1}{4}$ and $a_n=\frac{2^n}{n}$ since $\sum_{n=1}^{\infty}\frac{(4x-1)^n}{n2^n}=\sum_{n=1}^{\infty}\frac{4^n(x-\frac{1}{4})^n}{n2^n}$

for the radius

$\left|\frac{a_{n+1}}{a_n}\right|=\frac{2^{n+1}}{(n+1)}\cdot\frac{n}{2^n}=2\left(\frac{n}{n+1}\right)\to 2$ as $n\to\infty$, so $R=\frac{1}{2}$

and the series is **absolutely convergent** for $x\in\left(-\frac{1}{4},\frac{3}{4}\right)$

for $x=\frac{3}{4}$the series $\sum_{n=1}^{\infty}\frac{(4x-1)^n}{n2^n}=\sum_{n=1}^{\infty}\frac{(2)^n}{n2^n}=\sum_{n=1}^{\infty}\frac{1}{n}=\infty$ harm.series

and for $x = -\frac{1}{4}$ the series $\sum_{n=1}^{\infty} \frac{(4x-1)^n}{n2^n} = \sum_{n=1}^{\infty} \frac{(-2)^n}{n2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
which is **cond.convergent** by Alt.Test
Together the series is convergent for $x \in \left[-\frac{1}{4}, \frac{3}{4}\right)$

For 3)

first

$$\frac{1}{x} = \frac{1}{x+2-2} = -\frac{1}{2} \cdot \frac{1}{1-\frac{x+2}{2}} = -\frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x+2}{2}\right)^n = -\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^{n+1}}$$

for $-1 < \frac{x+2}{2} < 1,$ so $-2 < x+2 < 2$, and finally $-4 < x < 0$

(using $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ for $-1 < r < 1$)

now, differentiate

$$\frac{-1}{x^2} = -\sum_{n=0}^{\infty} \frac{n\,(x+2)^{n-1}}{2^{n+1}} \qquad \text{so } \frac{1}{x^2} = \sum_{n=0}^{\infty} \frac{n\,(x+2)^{n-1}}{2^{n+1}} = \sum_{k=1}^{\infty} \frac{(k+1)\,(x+2)^k}{2^{k+2}}$$

for $-4 < x < 0$.