Department of Mathematics and Statistics
MATH 349-02
Quiz # 3T **Fall 2006**

Name:____________________________**I.D.#:**_________________

1. Is the series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^2+16}$ absolutely or conditionally convergent or divergent? Explain. [3]

2. Find the centre,radius and interval of convergence of power series $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}}$. [4]

3. Expand $f(x) = \ln x$ in power series with the center at $c = 2$. Find the interval where the representation is valid. [3]

Solution

For 1)

first $\sum_{n=1}^{\infty} \left|(-1)^n \frac{n}{n^2+16}\right| = \sum_{n=1}^{\infty} \frac{n}{n^2+16}$

$$\frac{n}{n^2+16} \backsim \frac{n}{n^2} = \frac{1}{n} \text{ since } \frac{\frac{n}{n^2+16}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2}{n^2+16} = \lim_{n\to\infty} \frac{1}{1+\frac{16}{n^2}} = 1 \neq 0$$

and $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$ so by Limit Comp.Test $\sum_{n=1}^{\infty} \left|\frac{n}{n^2+16}\right| = \infty$

it is **cond.convergent** by Alt test since the sequence $a_n = \frac{n}{n^2+16}$ is decr and convergent to 0

$\lim_{n\to\infty} \frac{n}{n^2+16} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{16}{n^2}} = 0$ and for $f(x) = \frac{x}{x^2+16}$

$f'(x) = \left(\frac{x}{x^2+16}\right)' = \frac{x^2+16-2x^2}{(x^2+16)^2} = \frac{16-x^2}{(x^2+16)^2} < 0$

for $x > 4$ thus the sequence is ult.decr.

For 2)

The centre is $c = -\frac{1}{3}$ and $a_n = \frac{3^n}{\sqrt{n}}$ since $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n\left(x+\frac{1}{3}\right)^n}{\sqrt{n}}$

Since $\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1}}{\sqrt{(n+1)}} \cdot \frac{\sqrt{n}}{3^n} = 3 \cdot \sqrt{\frac{n}{n+1}} \to 3$ so $R = \frac{1}{3}$

and the series is **abs.convergent** on the interval is $\left(-\frac{2}{3}, 0\right)$.

now the ends

for $x = 0$ $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \infty$ since $p < 1$

$x = -\frac{2}{3}$ $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ it is **cond.convergent** since the sequence $\frac{1}{\sqrt{n}}$ is decr. $= \frac{1}{pos.incr.}$ and $\frac{1}{\sqrt{n}} \to 0$ together the series is convergent on $\left[-\frac{2}{3}, 0\right)$

For 3)

The answer must be in the form $\sum_{n=0}^{\infty} a_n (x-2)^n$ .Rewrite

$f(x) = \ln x = \ln(x - 2 + 2) = \ln 2(1 + \frac{x-2}{2}) = \ln 2 + \ln(1 + \frac{x-2}{2})$

$= \ln 2 + \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)\, 2^{n+1}} (x-2)^{n+1} = \ln 2 - \sum_{k=1}^{\infty} \frac{(-1)^k}{k 2^k} (x-2)^k$

for $-1 < \frac{(x-2)}{2} \leq 1$

$-2 < x - 2 \leq 2$ so $0 < x \leq 4$

using $\ln(1+s) = \sum_{n=0}^{\infty} (-1)^n \frac{s^{n+1}}{n+1}$ for $-1 < s \leq 1$ with $s = \frac{(x-2)}{2}$.

you can check $a_0 = f(2) = \ln 2$.