**The University of Calgary**
**Department of Mathematics and Statistics**
**MATH 349-02**
**Quiz # 4R**

**Fall,2006**

**I.D.#:**________________ **Name:**

1. For $f(x,y) = e^{\frac{y}{x}}$

   (a) sketch the domain of $f$;find the range;

   (b) sketch the level curves of $f$ for $c = 0, 1, -1, e$;

   (c) find an equation of the tangent plane to $z = f(x,y)$ at $x = -1, y = 0$. [7]

2. Find $\lim_{(x,y)\to(-1,0)} \frac{\sin(xy+y)}{2(x+1)^2+3y^2}$ , if it exists. [3]

**Solution For 1)**

domain :$x \neq 0$,the xy-plane without the y-axis

$c = e^{\frac{y}{x}}$ for $c \leq 0$ no curves;for $c > 0$

$\ln c = \frac{y}{x}$ $y = (\ln c)x$ LINES THROUGH THE ORIGIN WITHOUT IT

for any $c = 1$ the x-axis; for $c = e$ $y = x$

the range is $(0, +\infty)$

partials

$f_x(x,y) = e^{\frac{y}{x}} \left(\frac{-y}{x^2}\right)$ at $x = -1, y = 0$ $f_x = 0$

$f_y(x,y) = e^{\frac{y}{x}} \left(\frac{1}{x}\right)$ at $x = -1, y = 0$ $f_y = -1$

so a normal vector $\overrightarrow{n} = (\nabla f, -1) = (0, -1, -1)$ or$(0, 1, 1)$

for the point we need

$z_0 = f(-1, 0) = 1$ $P\,(-1, 0, 1)$

an equation is $0\,(x+1) + y + (z-1) = 0$ or $y + z = 1$

**For 2)**

define $f(x,y) = \frac{\sin{(xy+y)}}{2(x+1)^2+3y^2}$around the point $(-1,0)$

then for $y \neq 0, x = -1$ $f(-1,y) = \frac{0}{3y^2} = 0$

for $x \neq -1, y = 0$ $f(x,0) = \frac{0}{2(x+1)^2} = 0$

let's try a line through the point $(-1,0)$ $y = m(x+1)$ for $x \neq -1$ and $m \neq 0$

$$f(x, m(x+1)) = \frac{\sin{(m(x+1)^2)}}{2(x+1)^2 + 3m^2(x+1)^2} = \frac{\sin{(m(x+1)^2)}}{m(x+1)^2} \cdot \frac{m}{(2+3m^2)}$$

as $x \to -1$ $f(x, m(x+1)) \to \frac{m}{(2+3m^2)} \neq 0$ (e.g.for $m = 1$)

since $\lim_{h\to 0} \frac{\sin\ h}{h} = 1$ thus **the limit DNE-does not exist.**