The University of Calgary
Department of Mathematics and Statistics
MATH 349
Quiz # 5T

Fall,2006

Name:______________________________**I.D.#:**__________________

1. Using Chain Rule find $h'(\frac{1}{2})$ if $h(t) = f(x, y, z)$ and $f(x, y, z) = xy^2\sqrt{x^2 + y^2 + e^z + \frac{3}{2}}$ and $x = \sin(\pi t)$, $y = \cos(2\pi t)$ and $z = \ln t$. [5]

2. For the function $f(x, y) = e^{1-\frac{x}{y}}$ find the directional derivative $D_{\mathbf{u}}f$ at the point $A\,(2, 2)$ in the direction from A to $B\,(1, -1)$. Can you find a direction succh that $D_{\mathbf{u}}f\ (2, 2) = 1$?

   Explain. [5]

Solutions

For 1)

for $t =\frac{1}{2}$ $\quad x = \sin(\frac{\pi}{2}) = 1$, $y = \cos(\pi) = -1$ and $z = \ln\frac{1}{2} = -\ln 2$.

$$f_x = y^2\sqrt{x^2 + y^2 + e^z + \tfrac{3}{2}} + \frac{x^2y^2}{\sqrt{x^2 + y^2 + e^z + \frac{3}{2}}},$$

$$f_y = 2xy\sqrt{x^2 + y^2 + e^z + \tfrac{3}{2}} + \frac{xy^3}{\sqrt{x^2 + y^2 + e^z + \frac{3}{2}}}$$

$$f_z = \frac{xy^2e^z}{2\sqrt{x^2 + y^2 + e^z + \frac{3}{2}}} \text{ are cont ,and}$$

$\nabla f\,(1, -1, -\ln 2) = (f_x, f_y, f_z)$ at $(1, -1, -\ln 2) = \left(\frac{5}{2}, -\frac{9}{2}, \frac{1}{4}\right) = \frac{1}{4}\,(10, -18, 1)$

$(x', y', z') = \left(\pi\cos\pi t, , -2\pi\sin 2\pi t, \dfrac{1}{t}\right)$ at $t = \frac{1}{2}$ $\quad \left(x'\left(\frac{1}{2}\right), y'\left(\frac{1}{2}\right), z'\left(\frac{1}{2}\right)\right) = (0, 0, 2)$

$h'(\frac{1}{2}) = f_xx' + f_yy' + f_zz' = \nabla f\,(1, -1, -\ln 2) \bullet \left(x'\left(\frac{1}{2}\right), y'\left(\frac{1}{2}\right), z'\left(\frac{1}{2}\right)\right)$

$= \frac{1}{4}\,(10, -18, 1) \bullet (0, 0, 2) = \frac{1}{2}$

For 2)

partials $f_x = e^{1-\frac{x}{y}}\left(-\frac{1}{y}\right)$ $\quad f_y = e^{1-\frac{x}{y}}\left(\frac{x}{y^2}\right)$ are cont. for any x and $y \neq 0$

so we can use the theorem $D_{\mathbf{u}}f\ (2, 2) = \nabla f(2, 2) \bullet \mathbf{u}$ where $\overrightarrow{PR} = (-1, -3)$

$\mathbf{u} =\frac{-1}{\sqrt{10}}\,(1, 3)$ and $f_x(2, 2) = -\frac{1}{2}$ $\quad f_y(2, 2) = \frac{1}{2}$ $\quad \nabla f\,(2, 2) = \frac{1}{2}\,(-1, 1)$

thus

$D_{\mathbf{u}}f\ (2, 2) = \nabla f(2, 2) \bullet \mathbf{u} =\frac{-1}{2\sqrt{10}}\,(-1, 1)\cdot(1, 3) = -\dfrac{1}{\sqrt{10}}$

we know that max $D_{\mathbf{u}}f =\|\nabla f\| = \frac{1}{\sqrt{2}} < 1$ so it is impossible.