The University of Calgary
Department of Mathematics and Statistics
MATH 349-02
Quiz # 5R

Fall,2006

Name:______________________________**I.D.#:**__________________

1. Using Chain Rule find $\nabla h\,(4,-2)$ where $h(u,v)=f(x,y)$ with $x=\dfrac{u}{v^3}$ and $y=\dfrac{v^2}{u}$ and $f(x,y)=\ln\left(1+2x^2+xy^2\right).$ [5]

2. For the function $f(x,y)=x^2\sqrt{2+x^2+y^2}$ find the directional derivative $D_{\mathbf{u}}f$ at the point $A\,(-1,1)$
   in the direction of the vector $\mathbf{d}=(1,-3)$. Can you find a direction such that $D_{\mathbf{v}}f$ $(-1,1)=0$?
   Explain. [5]

   Solution

For 1)

if $u=4$ and $v=-2$ $\quad x=-\dfrac{1}{2}\quad y=1$

first we need $\nabla f=(f_x,f_v)=\left(\dfrac{4x+y^2}{1+2x^2+xy^2},\dfrac{2xy}{1+2x^2+xy^2}\right)$ cont. at $\left(-\frac{1}{2},1\right)$

then $\nabla f(-\frac{1}{2},1)=(-1,-1)$

and $\dfrac{\partial x}{\partial u}=\dfrac{1}{v^3}\quad \dfrac{\partial x}{\partial v}=\dfrac{-3u}{v^4}\quad \dfrac{\partial y}{\partial u}=\dfrac{-v^2}{u^2}\quad \dfrac{\partial y}{\partial v}=\dfrac{2v}{u}$

at $u=4,v=-2$ we get $\begin{bmatrix}\dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v}\\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v}\end{bmatrix}=\begin{bmatrix}-\frac{1}{8} & -\frac{3}{4}\\ -\frac{1}{4} & -1\end{bmatrix}$

then $\nabla h\,(4,-2)=\nabla f\left(-\frac{1}{2},1\right)D\mathbf{g}\,(4,-2)=[-1\ -1]\begin{bmatrix}-\frac{1}{8} & -\frac{3}{4}\\ -\frac{1}{4} & -1\end{bmatrix}=\begin{bmatrix}\dfrac{3}{8} & \dfrac{7}{4}\end{bmatrix}$

OR

$\dfrac{\partial h}{\partial u}(4,-2)=\dfrac{\partial f}{\partial x}\left(-\frac{1}{2},1\right)\bullet\dfrac{\partial x}{\partial u}(4,-2)+\dfrac{\partial f}{\partial y}\left(-\frac{1}{2},1\right)\bullet\dfrac{\partial y}{\partial u}(4,-2)=(-1,-1)\bullet\left(\frac{-1}{8},\frac{-1}{4}\right)=\dfrac{3}{8}$

$\dfrac{\partial h}{\partial v}(4,-2)=\dfrac{\partial f}{\partial x}\left(-\frac{1}{2},1\right)\bullet\dfrac{\partial x}{\partial v}(4,-2)+\dfrac{\partial f}{\partial y}\left(-\frac{1}{2},1\right)\bullet\dfrac{\partial y}{\partial v}(4,-2)=(-1,-1)\bullet\left(-\frac{3}{4},-1\right)=\dfrac{7}{4}$

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For 2)

$f_x=2x\sqrt{2+x^2+y^2}+\dfrac{x^3}{\sqrt{2+x^2+y^2}}\qquad f_y=\dfrac{x^2y}{\sqrt{2+x^2+y^2}}$ cont. everywhere

at $A\,(-1,1)\qquad \nabla f\ (-1,1)=\left(-\frac{9}{2},\frac{1}{2}\right)=\frac{1}{2}\,(-9,1)$

$\mathbf{u}=\frac{\mathbf{d}}{\|\mathbf{d}\|}=\frac{1}{\sqrt{10}}\,(1,-3)$ and by Theorem

$$D_{\mathbf{u}}f\ (-1,1)=\nabla f\ (-1,1)\bullet\mathbf{u}=\tfrac{1}{2\sqrt{10}}\,(-9,1)\bullet(1,-3)=-\frac{6}{\sqrt{10}};$$

For the rate $=0$ the vector $\mathbf{v}$ must be perpendicular to $\nabla f(-1,1)$ thus $\mathbf{v}=\pm\frac{1}{\sqrt{82}}\,(1,9)$.