The University of Calgary
Department of Mathematics and Statistics
MATH 349 Handout # 5

1. For $f(x,y)=\dfrac{xy}{\sqrt{1+x^2}}$ and $\mathbf{g}: R^2 \to R^2, \mathbf{g}(s,t)=(\cos(\pi st), \sin\dfrac{\pi s}{t})$
   (a) find ∇f;
   (b) using Chain Rule find $\nabla h(0,-1)$ where $h=f\circ\mathbf{g}$
   (or $h(s,t)=f(x,y)$ where $x=\cos(\pi st)$ and $y=\sin\dfrac{\pi s}{t}$)

2. Find an equation of the tangent plane to $z=f(x,y)=\ln(x+y^2)$ at the point $x_0=0, y_0=-1$.

3. For the function $f(x,y)=e^{\sqrt{\frac{y}{x}}}$ find the domain and f_{xx} and f_{xy}.

4. For $f(x,y,z)=\sqrt{2}\sin(\pi xy+x\ln z)$ and $\mathbf{g}: R\to R^3, \mathbf{g}(t)=(\frac{1}{t},-\frac{1}{t},\frac{t}{2})$
   (a) find ∇f;
   (b) find $D\mathbf{g}$ or $\mathbf{g}'$
   (c) using Chain Rule find $h'(2)$ where $h=f\circ\mathbf{g}$
   (or $h(t)=f(x,y,z)$ where $x=\frac{1}{t}$, $y=\frac{-1}{t}$ and $z=\frac{t}{2}$)

5. In what directions at the point $P(2,1)$ does the function $f(x,y)=\ln\left(\frac{x}{y}+\frac{y}{x}\right)$ have the rate of change equal to $\frac{3}{10}$?What is the maximum rate of change at that point?

6. Find the rate and the direction of the most rapid decrease of $f(x,y,z)=x^2ze^y+xz^2$ at the point$P(1,\ln 2,2)$.

7. Given $f(x,y)=\begin{cases}\dfrac{xy}{2x^2+y^2} \text{for } (x,y)\neq(0,0)\\ 0 \text{ ....at } (0,0)\end{cases}$
   (a) Is f continous at $(0,0)$?
   (b) Find ∇f at $(0,0)$, if it exists.
   (c) Find the directional derivative at $(0,0)$ in the direction of $y=x$ if it exists.
   (d) Find the directional derivative at $(-1,-1)$ where $\mathbf{u}=\dfrac{1}{\sqrt{2}}(1,1)$.